International Trade and Industrialization in a Non-Scale Model of Economic Growth

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Abstract. The relationship between international trade, growth, and industrialization is analyzed in a two-sector non-scale growth model. The counterfactual prediction of new growth theories regarding a positive effect of population growth on per capita income growth is shown to be alleviated by allowing for international trade. While the growth-trade linkage is positive in most cases, it is negative if the rate of population growth is relatively large and the initial capital stock is relatively small. As the timing of the switch from autarky to free trade affects the process of industrialization, trade policy can influence structural change and long-run growth rates even in non-scale growth models.
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1 Introduction

Among the important questions relating to international trade and economic growth is whether more openness of a country always increases its growth rate of per capita income or not. One of the most convincing arguments in favor of free trade respectively an outward-looking development policy is that international trade can act as an impetus for the flow of knowledge across international borders (aside from the static gains from trade). The most important argument in favor of an import substitution policy is the infant industry argument for protection. The new growth theory, which gives an endogenous explanation of growing labor productivity in the long run, provides the background for a more thorough theoretical analysis of these issues.

Jones (1995) pointed out, however, that *endogenous growth models* such as Romer's (1990) seminal contribution exaggerate by implying that an increase in the size or scale of an economy permanently increases its long-run growth rate of income per capita. This criticism of such *scale models* led to the formulation of *non-scale models* in which long-run per capita growth rates do not depend on pop-

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ulation (or another measure of scale) directly but just on the rate of population growth. As the long-run growth rates in non-scale models are usually independent of policy instruments, they are also known as models of *semi-endogenous growth*.

Although non-scale growth models need not be models of semi-endogenous growth, nor the other way around, it has been shown in Christiaans (2004) that in the absence of particular knife-edge conditions both properties appear together. Moreover, any model of steady state growth implies that a knife-edge condition must be satisfied (cf. Christiaans, 2004; Growiec, 2007). One such condition that appears to be justifiable is that population grows at a constant rate (Jones, 2001). Any model requiring no further knife-edge conditions must be of the semi-endogenous non-scale type considered in this paper.

The general steady state properties of such non-scale growth models have been analyzed by Eicher and Turnovsky (1999b) for the case of closed economies. It is straightforward that these models are reasonably consistent with the well known stylized facts about growth with two exceptions. First, a closed economy model obviously cannot address the empirically observed positive correlation between the growth in the volume of international trade and the growth of output (the *growth-trade linkage*). Second, the models are inconsistent with the observed negative correlation between population growth rates and the levels of per capita income (the *population puzzle*). Even though there is no unanimity regarding both of these facts in the literature, a number of cross-country studies have shown that population growth and growth of per capita output are either uncorrelated or even negatively correlated (e.g. Mankiw et al., 1992). While most of the empirical literature finds evidence for a positive growth-trade linkage, Rodriguez and Rodrik (2001) expressed serious scepticism about these results.

As has been shown in Christiaans (2003), the population puzzle need not arise in open economy models. That approach cannot at the same time address the growth-trade linkage, however. It is the purpose of the present paper to present a simple two-sector (manufacturing and agriculture) small open economy non-scale growth model dealing with the growth-trade linkage and at the same time alleviating the strong prediction that an increase in the rate of population growth *always* increases per capita income growth.

As Grossman and Helpman (1991) and Feenstra (1996) have stressed, the literature on trade and growth entails two opposite sets of results, which are related to the discussion about outward-looking versus import substitution policies. While models based on learning by doing or human capital accumulation (e.g. Lucas, 1988, sec. 5) usually predict unequal per capita income growth rates of economies (possible exceptions are outlined in Young, 1991; Goh and Olivier, 2002), models of endogenous technological change following Romer (1990) usually establish that trade will lead to a convergence of growth rates across countries. The latter result often depends on the crucial assumption that the international diffusion of knowledge appears simultaneously with trade. As Feenstra (1996) illustrates using a model in the tradition of Grossman and Helpman (1991), unequal growth is possible if there is no international diffusion of knowledge.

The empirical question of whether knowledge spillovers are primarily intranational or international in scope is therefore important for an assessment of international trade and growth. By way of example, the studies of Jaffe et al. (1993) and Branstetter (2001) provide evidence for geographically localized or intranational spillovers, respectively. Of course, these results do not imply that there is no international knowledge diffusion, but just that intranational diffusion is more important. According to Griffith et al. (2004), R&D does also enhance the absorptive capacity for inventions made abroad and therefore helps countries to "catch-up". Although Keller (2004) in his survey of international technology diffusion provides examples of countries for which foreign R&D matters even more than domestic R&D, he finds that the evidence generally supports the hypothesis that "technology diffusion within countries is stronger than across countries" (p. 772).

Given the ambivalence of empirical evidence, it is worthwhile to consider models with international as well as models with intranational knowledge spillovers as benchmark cases. The present model uses learning by investment as the engine of growth and sticks to the assumption that learning is external on the firm level but internal on the country level. Related to this assumption, the model also abstracts from international capital movements. In contrast to former scale models, it is possible to consider a positive rate of population growth in this non-scale model. The growth rates under free trade of the home country as compared to those of the rest of the world will be shown to depend on the relative magnitude of the domestic population growth rate (as compared to the average rate in the rest of the world) and the pattern of specialization.

In accordance with the empirical results of Rodriguez and Rodrik (2001), however, there is no unambiguous relationship between trade and growth. The reason is that switchovers in comparative advantages are possible.[1] Depending on the emerging pattern of specialization, there may be a positive or a negative growth-trade linkage. Moreover, it is possible that the growth rate of per capita income temporarily increases when switching to free trade as a result of accelerated industrialization, which in the long run is not sustainable, however. In such a case the eventually deindustrialized country will nevertheless gain from a relatively high growth rate abroad since its terms of trade will improve relatively fast. A negative linkage occurs if the population growth rate is relatively high and the initial capital stock is relatively small, in which case the home country has an initial comparative advantage in agricultural production. This result can partly explain why LDC's often have relatively high growth rates of population but relatively low growth rates of per capita income.[2]

The government cannot influence the long-run growth rates of per capita income in non-scale growth models of closed economies. International trade, which may affect this policy ineffectiveness, has so far largely been neglected in non-scale models. Among the few exceptions are Eicher and Turnovsky (1999a), Dinopoulos and Segerstrom (1999), and Arnold (2007). As Eicher and Turnovsky (1999a) discuss international capital flows in a one-sector non-scale growth model, there

[1] Cf. Landesmann and Stehrer (2001) for empirical evidence on switchovers in comparative advantages at the industrial level.

[2] Notice that the present model analyzes the effects of exogenous population growth rates on growth and comparative advantages. As to the reverse effect of initial comparative advantages on endogenous population and per capita income growth, cf. Galor and Mountford (2006).

is no possibility to explain international trade by comparative advantages. The two-country model of Dinopoulos and Segerstrom (1999) is concerned with a trade explanation of increasing wage inequality but not with different growth rates in the two economies, although international trade may affect steady state growth rates. Their model is far too much involved to allow for an analysis out of the steady state. Arnold (2007) considers a very general model that comprises several standard models of endogenous and semi-endogenous growth. However, his model includes just one homogeneous final good, and he concludes that in this class of models economic integration does not influence long-run growth rates in the semi-endogenous case.

Whenever trade policy influences structural change and the pattern of international specialization, however, trade policy may at the same time be growth policy. Although the present model is not concerned with the derivation of optimum policy measures, it shows that on principle policy ineffectiveness with respect to long-run growth rates does not hold in open economy models of non-scale growth. Thus, under particular circumstances, an infant industry protection may be justified.

The paper is related to previous work by Matsuyama (1992) and Wong and Yip (1999). Matsuyama (1992) analyzes the effects of agricultural productivity on the growth rate of GNP using a non-homothetic utility function. Wong and Yip (1999) use a homothetic function but introduce the accumulation of capital, neglected by Matsuyama (1992). The present model also considers capital accumulation and simplifies matters by using homothetic preferences. It will equally be assumed that a small country faces a given growth rate of the rest of the world. The most important difference is that decreasing returns to knowledge accumulation are assumed, which turns the scale model of Wong and Yip (1999) into a non-scale growth model. Moreover, some simplifying assumptions like taking the capital stock as the learning index (dropping cumulated output as a state variable) enable a full-fledged dynamical analysis that is not confined to the steady state.

The basic closed economy version of the model is presented in Section 2, while Section 3 analyzes the possible steady states and the transitional dynamics of a small economy under free trade with the rest of the world. Longer derivations are relegated to appendices. The final section discusses the main results, their dependence on some of the assumptions, and their policy implications.

2 The Closed Economy

2.1 The Static Equilibrium

Each of a large number of completely identical firms j of sector 1 (manufacturing) uses labor L_{1j} and capital K_j to produce its output Y_{1j} according to the production function

$$Y_{1j} = K_j^{\alpha}(K^{\beta/(1-\alpha)}L_{1j})^{1-\alpha}, \quad 0 < \alpha, \beta < 1, \quad \alpha + \beta < 1.$$

The aggregate quantities are given by $L_1 = \sum_j L_{1j}$, $K = \sum_j K_j$, and $Y_1 = \sum_j Y_{1j}$. The presence of the aggregate capital stock K implies labor-augmenting technical progress akin to the learning by doing respectively learning by investment formulation of

Sheshinski (1967) or the endogenous growth model of Romer (1986). Notice that the exponent of K, $\beta/(1-\alpha)$, is smaller than one due to the assumption that $\alpha+\beta<1$. If $1-\alpha-\beta=0$, the model would involve scale-effects (cf. Jones, 1999; Christiaans, 2004). Although the individual production functions will not be used in the sequel, they are discussed here in order to emphasize that an external effect of learning by investment is assumed. If learning was internal to firms, perfect competition would be impossible.

Under perfect competition, the individual production functions j can be aggregated to a sectoral production function for the manufactured commodity 1 (cf. e.g. Sargent, 1987, p. 10):

$$Y_1 = K^{\alpha}(K^{\beta/(1-\alpha)}L_1)^{1-\alpha} = K^{\alpha+\beta}L_1^{1-\alpha}, \quad 0<\alpha,\beta<1,\ \alpha+\beta<1. \tag{1}$$

While physical capital, K, is exclusively used in sector 1, labor, L, is allocated between both sectors with L_i denoting the amount of labor employed in sector i ($i=1,2$). Production in the second sector (agriculture) uses no capital and is linear in labor. To simplify the discussion of different learning opportunities across sectors as much as possible, the input coefficient of labor in sector 2 is constant and normalized to one. Thus, the production function of sector 2 can be written as

$$Y_2 = L_2. \tag{2}$$

Of course, Y_i, K, L, and L_i depend on time, but for the sake of notational convenience the time index t has been dropped. The supposed production structure constitutes a minimal approach to a factor endowments theory of international trade in which a weak version of Rybczynski's theorem holds [cf. (3) and (4) below].

From the full employment condition for labor, $L_1+L_2=L$, and equation (2) one gets $L_1 = L-Y_2$, where it is understood that all variables are not negative. Substituting into (1) yields the transformation frontier at any point in time. As is well known in international trade theory, the supply functions in a setting of perfect competition are the solutions of the revenue maximization problem[3]

$$\max_{Y_1\geqq 0,\, Y_2\geqq 0} \left\{pY_1+Y_2 \mid Y_1 = K^{\alpha+\beta}(L-Y_2)^{1-\alpha}\right\},$$

where p denotes the relative price of commodity 1 in terms of commodity 2, which is taken as the numéraire. The implied supply functions are

$$Y_1 = (1-\alpha)^{(1-\alpha)/\alpha}K^{(\alpha+\beta)/\alpha}p^{(1-\alpha)/\alpha} \tag{3}$$

$$Y_2 = L-(1-\alpha)^{1/\alpha}K^{(\alpha+\beta)/\alpha}p^{1/\alpha} \tag{4}$$

[3]Notice that the transformation frontier is strictly concave. Since physical capital is exclusively used in sector 1, static revenue maximization is determined by the allocation of labor to manufacturing and agriculture. Thus, the dynamic externality of learning by investment has no distorting impact on static revenue maximization, although the private and social marginal productivities of capital do not coincide. Given the existing stock of capital at any instant, equating the marginal productivities of labor in both sectors by profit-maximizing individual firms would also yield the supply functions (3) and (4). Thus, although there is an externality of capital in the manufacturing sector, perfectly competitive firms will maximize national income in the short run.

Complete specialization in agriculture is not possible as long as $0 < p < \infty$ and $K > 0$. The economy will completely specialize in manufacturing, however, if p or K take on sufficiently high values.

Households decide about saving and the allocation of consumption with respect to the two commodities. As a simple saving hypothesis, an extreme version of the classical saving function is assumed. Accordingly, all capital income is saved and all labor income is consumed. This assumption appears to be reasonable in positive non-scale growth theory, where it should be noted that the steady state growth rates are largely independent of the particular consumption hypothesis. Moreover, the classical saving function involves some degree of rationality since in its extreme version it implies convergence to the golden rule path in the absence of externalities (cf. Burmeister, 1980, p. 60). It may therefore be interpreted as a simple rule of thumb for consumers out of their depth with dynamic optimization methods.

The instantaneous utility function is assumed to be of the Cobb-Douglas form $U(C_1, C_2) = C_1^{\theta_1} C_2^{1-\theta_1}$, where C_i denotes consumption of commodity i, $i = 1, 2$, and $0 < \theta_1 < 1$. This utility function implies that both commodities are consumed and therefore produced in a closed economy (if feasible). Hence, it follows from equation (2) that the wage rate in terms of the second commodity is given by $w = \partial Y_2 / \partial L_2 = 1$. Therefore, the representative household at any point in time maximizes his utility function subject to the constraint $pC_1 + C_2 = L$. The solution to this problem is

$$pC_1 = \theta_1 L, \tag{5}$$

$$C_2 = (1 - \theta_1) L. \tag{6}$$

While it is assumed that the agricultural product is a pure consumption good, the manufactured good shall serve for consumption as well as for investment. Thus, the overall demand for the first commodity comprises C_1 and investment demand.

Equating demand, (6), and supply, (4), of the second commodity yields the relative price in short-run equilibrium:

$$p = \frac{\theta_1^{\alpha} L^{\alpha}}{(1-\alpha) K^{\alpha+\beta}}. \tag{7}$$

Notice that the variables L and K are predetermined at any point in time. Substituting into the supply functions (3) and (4), respectively, leads to the equilibrium outputs of both sectors as functions of the predetermined variables:

$$Y_1 = \theta_1^{1-\alpha} K^{\alpha+\beta} L^{1-\alpha} \tag{8}$$

$$Y_2 = (1 - \theta_1) L \tag{9}$$

As can be seen from these equations, production will be diversified in the closed economy (both goods are produced).

2.2 Dynamics

The population equals the labor force and grows at an exogenous and constant rate n, $0 < n < 1$, that is $g_L := \dot{L}/L = n$. (In general, derivatives with respect to time are indicated by a dot and the growth rate of any variable x is denoted as g_x.) As the manufactured good can both be consumed and invested, aggregate gross investment, I, in short run equilibrium is given by the difference between output, Y_1, and consumption, C_1, of the first commodity. Neglecting the depreciation of capital for simplicity, gross investment equals net investment:

$$\dot{K} = I = Y_1 - C_1. \tag{10}$$

A steady state growth path or long-run equilibrium is defined as a path along which all variables grow at constant rates. Substituting (7) into (5) and dividing by Y_1 according to (8) implies $C_1 = (1-\alpha)Y_1$. Inserting into (10) and dividing by K yields $g_K = \alpha Y_1/K$. Therefore, the capital-output-ratio in sector 1 must be constant in a steady state ($g_{Y_1} = g_K$). Imposing the conditions of steady state growth and using $g_L = n$, logarithmic differentiation of equations (8), (9), and (7), respectively, yields the following growth rates in a long-run equilibrium:

$$g_{Y_1} = g_K = \underbrace{\frac{1-\alpha}{1-\alpha-\beta}}_{=:\gamma} n = \gamma n, \tag{11}$$

$$g_{Y_2} = n, \tag{12}$$

$$g_p = -\frac{\beta}{1-\alpha-\beta} n = (1-\gamma)n. \tag{13}$$

Since it has been assumed that $1-\alpha-\beta > 0$, it follows that $\gamma > 1$. Thus, the output of the first commodity grows faster than labor and the model generates semi-endogenous per capita growth. Using equations (11)–(13), the steady state growth rate of national income in terms of the second commodity, $Y = pY_1 + Y_2$, is

$$g_Y = (g_p + g_{Y_1})\frac{pY_1}{Y} + g_{Y_2}\frac{Y_2}{Y} = n.$$

While this expression seems to imply that there is no long-run growth of per capita income, $g_{Y/L} = 0$, it must be noted that this result does not take into account the steady decline of the relative price of the first commodity. Thus, the growth rate of national income (per capita) in terms of manufactured goods is

$$g_{Y/p} = g_Y - g_p = \gamma n, \; g_{Y/(pL)} = (\gamma - 1)n > 0.$$

In a steady state, $g_{Y_1} - \gamma g_L = g_K - \gamma g_L = 0$ and $g_{Y_2} - g_L = 0$, which implies that the following *scale adjusted* per capita variables are constant in long-run equilibrium:

$$y_1 := \frac{Y_1}{L^\gamma}, \; k := \frac{K}{L^\gamma}, \; y_2 := \frac{Y_2}{L}. \tag{14}$$

As shown in Appendix A, the model can now be reduced to a differential equation in k describing the dynamic behavior of the closed economy:

$$\dot{k} = \alpha\theta_1^{1-\alpha}k^{\alpha+\beta} - \gamma n k. \tag{15}$$

Since $0 < \alpha + \beta < 1$ has been assumed, it is straightforward that equation (15) possesses a unique positive equilibrium value

$$k_e^a = \left(\frac{\alpha \theta_1^{1-\alpha}}{\gamma n} \right)^{1/(1-\alpha-\beta)} \tag{16}$$

which is globally stable for any historically given initial value of the scale adjusted per capita stock of capital, $k(0) = k_0 > 0$. It is therefore reasonable to consider the steady state as describing the long-run development of the closed economy.

The implications of this model with respect to the comparison of various autarkic countries are much like those of other standard models of non-scale growth. The following proposition summarizes the main results.

Proposition 1 *If $k_0 > 0$, there is a unique, globally stable steady state under autarky. In long-run equilibrium, production is diversified and the output of agricultural goods grows at the rate n. The output of manufactured goods grows at the rate $\gamma n > n$ and the relative price of manufactured goods decreases at the rate $(1-\gamma)n$. National income per capita in terms of manufactured goods grows at the rate $(\gamma - 1)n > 0$. It is constant in terms of agricultural goods.*

3 The Small Open Economy

3.1 Steady State Analysis

Diversification In the case of a small open economy under free trade with the rest of the world (ROW), the time path of the relative price p of the manufactured commodity in terms of the agricultural good is exogenously given by the world market. Accordingly, p is from now on interpreted as being *exogenous to the home country*. The allocation of consumption depends on p, and supply and demand need not be equalized in the domestic markets. With respect to the dynamic behavior of the model, it is sufficient to consider the households' saving decision. The supply functions are (3) and (4) as before.

It will be assumed that all parameters of the ROW are equal to the corresponding parameters of the home country, except for the rate of population growth, which may be $n^* \neq n$. This assumption, which is analogous to the corresponding assumptions in Matsuyama (1992) and Wong and Yip (1999), implies that all steady state growth rates derived in Section 2 may now be reinterpreted as being the growth rates of the ROW, exogenous to the home economy.[4] Supposing that the ROW is in a steady state, the exogenous growth rate of the price ratio is thus

$$g_p = -\frac{\beta}{1-\alpha-\beta} n^* = (1-\gamma) n^* \tag{17}$$

For ease of reference, the assumptions on which all of the following propositions are based are summarized: *The home country produces according to (1), (2), and*

[4] If n exceeded n^*, the home country would necessarily become a large country in the long run. It should be noted, however, that n^* has to be interpreted as the average growth rate of the ROW, and the ROW consists of a large number of countries. Thus, the assumption of a small country may be justified for a relatively long time horizon.

$L_1 + L_2 = L$ in a setting of perfect competition, the population growth rate being $g_L = n$. Both commodities are consumed, while the first commodity serves also as an investment good. All capital income is saved and all labor income is consumed. The ROW is on a steady state path and the price ratio declines at the rate $g_p = (1-\gamma)n^*$, exogenous to the home country.

As long as production is diversified, the wage rate is $w = 1$ and aggregate consumption satisfies $pC_1 + C_2 = L$. The saving-investment decision under the extreme classical saving hypothesis is therefore determined by

$$pI = p\dot{K} = pY_1 + Y_2 - L,$$

which together with (3) and (4) yields

$$\dot{K} = \alpha(1-\alpha)^{(1-\alpha)/\alpha} K^{(\alpha+\beta)/\alpha} p^{(1-\alpha)/\alpha}. \tag{18}$$

Note that investment out of imported manufactured goods is possible now.

As the ROW is in a steady state, logarithmic differentiation of (18) using (17) implies that a constant growth rate of K requires $g_{\dot{K}} = g_K = [(\alpha+\beta)/\alpha]g_K + [(1-\alpha)/\alpha](1-\gamma)n^*$ and therefore

$$g_K = -\frac{1-\alpha}{\beta} g_p = \gamma n^*.$$

Hence, as $g_K + [(1-\alpha)/\beta]g_p = 0$, the variable $\tilde{k}$ defined by

$$\tilde{k} := Kp^{(1-\alpha)/\beta} \tag{19}$$

must be constant in a steady state and the dynamics in case of diversification can be analyzed in terms of $\tilde{k}$. From the definition of $\tilde{k}$ and (17), $\dot{\tilde{k}} = g_K\tilde{k} - \gamma n^*\tilde{k}$. Substitution of g_K according to (18) (out of the steady state) and noting that $K^{\beta/\alpha}p^{(1-\alpha)/\alpha} = \tilde{k}^{\beta/\alpha}$ yields

$$\dot{\tilde{k}} = \alpha(1-\alpha)^{(1-\alpha)/\alpha}\tilde{k}^{(\alpha+\beta)/\alpha} - \gamma n^*\tilde{k}. \tag{20}$$

The equilibrium solution of this equation is

$$\tilde{k}_e = \left(\frac{\gamma n^*}{\alpha(1-\alpha)^{(1-\alpha)/\alpha}}\right)^{\alpha/\beta}. \tag{21}$$

Since $(\alpha+\beta)/\alpha > 1$, the long-run equilibrium $\tilde{k}_e$ is unstable (cf. Figure 1).[5] Moreover, the existence of a steady state (for all variables) further requires that $n = n^*$. For a proof, a procedure similar to the one used in deriving equation (15) (cf. Appendix A) may be applied in order to express equations (3) and (4) as functions of $\tilde{k}$:

$$Y_1 = (1-\alpha)^{(1-\alpha)/\alpha}\tilde{k}^{(\alpha+\beta)/\alpha}p^{-(1-\alpha)/\beta}, \tag{22}$$

$$Y_2 = L - (1-\alpha)^{1/\alpha}\tilde{k}^{(\alpha+\beta)/\alpha}p^{-(1-\alpha-\beta)/\beta}. \tag{23}$$

[5] Equation (20) is valid for $t \to \infty$ only if $\tilde{k}(0) = \tilde{k}_0 \leqq \tilde{k}_e$. If $\tilde{k}_0 > \tilde{k}_e$, there is a finite $\bar{t}$ such that $\lim_{t\to\bar{t}} \tilde{k}(t) = \infty$, cf. Appendix B. In such a case, the economy will switch to complete specialization in the production of the first commodity at a $t_1 < \bar{t}$. The dynamic evolution then follows another differential equation. If $\tilde{k}_0 < \tilde{k}_e$, it is proven in Appendix D that the economy asymptotically specializes in agriculture. Since $\tilde{k}(t) > 0$ and hence $K(t) > 0$ for any finite t, however, a small amount of the first commodity will nevertheless be produced and equation (20) continues to be valid.

Logarithmic differentiation of (22) and substitution of (17) for g_p implies that $g_{Y_1} = \gamma n^*$ in a steady state. Calculating the steady state growth rate for equation (23), however, implies

$$g_{Y_2} = n\frac{L}{Y_2} - n^*\frac{L - Y_2}{Y_2}. \tag{24}$$

Thus, g_{Y_2} would be constant only if either $n = n^*$, in which case $g_{Y_2} = n$, or if L/Y_2 was constant. The second case implies a contradiction since $g_{Y_2} = n$ from $L/Y_2 =$ const., while $g_{Y_2} \neq n$ from (24) if $n \neq n^*$.[6]

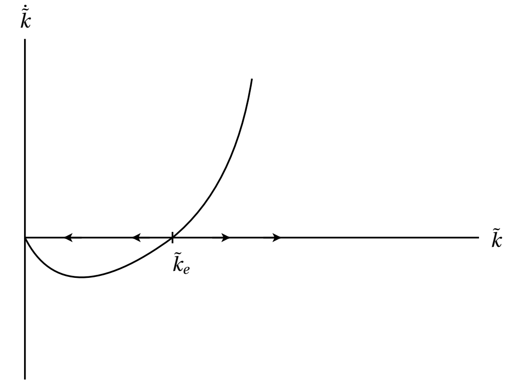

Figure 1. Instability of the Diversified Steady State

In view of these results, diversified production in the long run is a knife-edge event for the domestic economy. If the knife-edge conditions are not met, a process of complete industrialization or deindustrialization will be initiated at the time of switching to free trade. For the sake of completeness, notice that if $n = n^*$, (17), (22), and (23) immediately imply the final statement of the following proposition, which summarizes the main results.

Proposition 2 *Any diversified steady state under free trade is unstable and it does not exist unless the population growth rates at home and in the ROW coincide. If it exists and the home country is in its steady state when switching to free trade, all domestic growth rates coincide with the autarkic growth rates of Proposition 1.*

Comparative Advantages According to (7), the autarky price ratio depends on the absolute values of L and K as well as on the preference parameter θ_1. Thus, a general proposition in terms of the variables k and $\tilde{k}$ about the comparative advantages at the time when the country opens up to international trade is not possible. Let $\tilde{k}_0$ be the initial value of $\tilde{k}$ when the country opens up to international trade. E.g., $\tilde{k}_0 > \tilde{k}_e$ does not generally imply that the home country has a comparative

[6] The situation resembles the dynamic Oniki-Uzawa-Bardhan version of the $2 \times 2 \times 2$-model of international trade, where it is impossible for both countries to be on a steady state path unless the population growth rates coincide, cf. Bardhan (1970, p. 53). Although Khang (1971) has proven that, under suitable assumptions, both countries nevertheless asymptotically approach a steady state path, his result hinges on the fact that the relative population size of the country with the larger population growth rate converges to one while its per capita imports approach zero. In effect, the larger country is asymptotically autarkic, while the smaller country asymptotically completely specializes in such a case.

advantage in manufacturing. The following proposition is proven in Appendix C, however.

Proposition 3 *Suppose that the home country is on its autarkic steady state path at time $t = 0$ ($k_0 = k_e^a$) when it switches to free trade.*

- *If $n = n^*$, the home country has a comparative advantage in manufacturing (agriculture) if $\tilde{k}_0 > \tilde{k}_e$ ($\tilde{k}_0 < \tilde{k}_e$), while no international trade actually occurs if $\tilde{k}_0 = \tilde{k}_e$.*
- *If $n < n^*$, the home country has a comparative advantage in manufacturing if $\tilde{k}_0 \geqq \tilde{k}_e$. Otherwise, no simple statement is possible.*
- *If $n > n^*$, the home country has a comparative advantage in agriculture if $\tilde{k}_0 \leqq \tilde{k}_e$. Otherwise, no simple statement is possible.*

Proposition 3 together with Proposition 2 implies that actually no international trade takes place at a diversified steady state.

Specialization in Agriculture To be brief, the expression *specialization* will always mean *complete specialization* in the sequel. It is intuitively clear from Figure 1 and rigorously proven in Appendix D that the economy asymptotically specializes in agriculture if $\tilde{k}_0 < \tilde{k}_e$. If only the second commodity is produced, the relevant growth rates follow immediately from $Y = Y_2 = L$ and (17):

$$g_Y = g_{Y_2} = n,\ g_{Y/p} = n + (\gamma - 1)n^*,\ g_{Y/L} = 0,\ g_{Y/(pL)} = (\gamma - 1)n^*.$$

Thus, the per capita growth rate of the home economy is determined by the growth rate n^* of world population and independent of the own rate of population growth. Of course, since both commodities are consumed at home, the country imports manufactured goods and exports agricultural goods.

A steady state with specialization in agriculture is sustainable since p continuously declines, that is, the terms of trade of the country exporting agricultural goods steadily improve while there is no increase in productivity of manufacturing at home. In summary:

Proposition 4 *If $\tilde{k}_0 < \tilde{k}_e$, the home country asymptotically specializes in agriculture. Its steady state growth rate of per capita income is independent of its own rate of population growth. Per capita income in terms of manufactured goods grows at the rate $(\gamma - 1)n^* > 0$, while it is constant in terms of agricultural goods.*

Suppose that $n > n^*$. Then the home economy could reach a higher growth rate, $(\gamma - 1)n$, of per capita income by switching to autarky (if $K > 0$). Thus, if $n > n^*$ and $\tilde{k}_0 < \tilde{k}_e$, there is a negative growth-trade linkage related to a process of deindustrialization. As will soon be seen, there is a positive growth-trade linkage if $n > n^*$ and $\tilde{k}_0 > \tilde{k}_e$, related to a process of industrialization. In other words, a country with a higher growth rate of population than the ROW and $\tilde{k}_0 < \tilde{k}_e$ would be better-off by sticking to autarky and waiting till its capital stock has grown to ensure that $\tilde{k}_0 > \tilde{k}_e$ (recall equation (19) and note that capital in autarky grows faster than

$p^{(1-\alpha)/\beta}$ falls in the ROW if $n > n^*$). It is important to observe that, starting at the autarkic steady state, a comparative advantage in agriculture is necessary but not sufficient for this scenario (cf. Proposition 3).

Although it will be shown that a country will asymptotically specialize in agriculture regardless of the initial value $\tilde{k}_0$ if $n < n^*$, there is a positive growth-trade linkage in this case. According to Proposition 4, the growth rate of per capita income in terms of manufactured goods is $(\gamma - 1)n^*$, which exceeds the autarkic growth rate $(\gamma - 1)n$ according to Proposition 1 if $n < n^*$.

Specialization in Manufacturing Since only good 1 is produced in this case, the wage rate is no longer equal to $\partial Y_2/\partial L_2 = 1$, but is given by $w = p\partial Y_1/\partial L$, implying

$$wL = (1-\alpha)pY_1.$$

Substituting into $p\dot{K} = pY_1 - wL$ yields

$$\dot{K} = \alpha Y_1 = \alpha K^{\alpha+\beta} L^{1-\alpha}, \quad g_K = \alpha k^{\alpha+\beta-1}. \tag{25}$$

Inserting g_K into $\dot{k} = g_K k - \gamma n k$ from the definition of $k = K/L^\gamma$ implies

$$\dot{k} = \alpha k^{\alpha+\beta} - \gamma n k. \tag{26}$$

It follows from this equation that there is a unique and stable long-run equilibrium k_e provided that specialization in manufacturing is sustainable:

$$k_e = \left(\frac{\alpha}{\gamma n}\right)^{1/(1-\alpha-\beta)} \tag{27}$$

It is shown in Appendix B for the case of diversification that $\lim \tilde{k}(t) \to \infty$ in finite time if $\tilde{k}_0 > \tilde{k}_e$. Thus, since p declines, the right hand side of (23) implies that $Y_2(t_1) = 0$ at a finite time t_1 and specialization in manufacturing emerges. Since $L_1 = L$, it follows from equation (1) that $g_{Y_1} = g_K = \gamma n$ in a steady state. Together with (17) this result yields

$$g_Y = \gamma n + (1-\gamma)n^*, \; g_{Y/L} = (\gamma - 1)(n - n^*), \; g_{Y/p} = \gamma n, \; g_{Y/(pL)} = (\gamma - 1)n.$$

Of course, the country exports manufactured goods in exchange for agricultural goods.

While a thorough analysis of the transitional dynamics is postponed to Section 3.2, it may be noted at this stage that the steady state with specialization in manufacturing is sustainable only if $n \geqq n^*$. To prove this, note that the right hand side of equation (4) must remain non-positive, which requires

$$n \leqq \frac{\alpha+\beta}{\alpha} g_K + \frac{1}{\alpha} g_p = \frac{\alpha+\beta}{\alpha} \frac{1-\alpha}{1-\alpha-\beta} n - \frac{\beta}{\alpha(1-\alpha-\beta)} n^* \iff n \geqq n^*.$$

In summary:

Proposition 5 *If $\tilde{k}_0 > \tilde{k}_e$, there is a finite time t_1 such that the home country specializes in manufacturing at t_1. This pattern of specialization is not sustainable unless $n \geqq n^*$. In this case, there is a unique and stable steady state in which the growth rate of per capita income increases in the domestic rate of population growth. The growth rate of per capita income in terms of agricultural goods is $(\gamma - 1)(n - n^*)$, while in terms of manufactured goods it grows at the rate $(\gamma - 1)n$.*

In contrast to the case of an economy specializing in agriculture, the faster the domestic population of an industrialized country grows, the higher is the growth rate of per capita income. In contrast to autarky, per capita income now also grows at a positive rate in terms of agricultural goods if $n > n^*$. This result points to an important source of dynamic gains from trade: If the ROW grows slower than the home country, the domestic terms of trade under free trade fall at a smaller rate than in autarky. Thus, if $n \geqq n^*$ and $\tilde{k}_0 \geqq \tilde{k}_e$, there is a non-negative growth-trade linkage. Moreover, if $n > n^*$, the growth rate of income per capita in terms of manufactured or agricultural goods according to Proposition 5 exceeds the corresponding growth rates of an agricultural country according to Proposition 4 despite the fact that the terms of trade of the industrialized country worsen steadily.

According to Proposition 3 it is possible that if $n > n^*$, a country has a comparative disadvantage in manufacturing even if $\tilde{k}_0 > \tilde{k}_e$. Under free trade, this disadvantage will switch over to an advantage and the country will grow faster than the ROW in the long run. Thus, initial backwardness may be overcome even under free trade if the country has the potential to grow fast, here measured by its relative population growth rate. The comparison of the various possible configurations that have been analyzed (and additionally the knife-edge cases that have been passed over) together with Proposition 3 yields the following

Proposition 6 *A negative influence of international trade on per capita income growth occurs if and only if $n > n^*$ and $\tilde{k}_0 \leqq \tilde{k}_e$. If the home economy starts at its autarkic steady state, an initial comparative advantage in agriculture is a necessary condition for a negative growth-trade linkage.*

3.2 Transitional Dynamics

The Dynamical System According to Proposition 5, it is possible that an economy with diversified production under free trade reaches specialization in manufacturing although this pattern of specialization will not be sustainable if $n < n^*$. This result suggests the importance of a thorough analysis of the transitional dynamics. As the dynamics in case of diversification or asymptotic specialization in agriculture are best described by the variable $\tilde{k}$ while k is the proper alternative in case of specialization in manufacturing, the analysis of phase diagrams in $(k, \tilde{k})$-space suggests itself.

The following results are proven in Appendix E. The transition line between the regions of diversification and specialization in manufacturing in $(k, \tilde{k})$-space is a hyperbola. Below this hyperbola, the economy is diversified as long as $(k, \tilde{k}) > (0, 0)$, and above it specializes in manufacturing. If the economy is diversified, both isoclines $\dot{k} = 0$ and $\dot{\tilde{k}} = 0$ are horizontal lines defined by $\tilde{k} = \tilde{k}_I$ and $\tilde{k} = \tilde{k}_e$ [cf. (21)], respectively, where $\tilde{k}_I$ is just a constant defined in Appendix E. In case of specialization in manufacturing, the isoclines are vertical lines defined by $k = k_e$ [cf. (27)] and $k = k_I$, respectively, where the constant k_I is again defined in Appendix E. The isoclines coincide if $n = n^*$, while $\dot{\tilde{k}} = 0$ lies above (below) $\dot{k} = 0$ if $n < n^*$ $(n > n^*)$. Thus there are three principal cases to consider. Finally, the assumption $0 < \theta_1 < 1$ implies that k_e according to (27) exceeds the autarkic equilibrium value k_e^a of k defined in (16). As it is not reasonable that an economy would start with $k > k_e^a$ when switching to free trade, only initial values $k_0 < k_e$ will be considered.

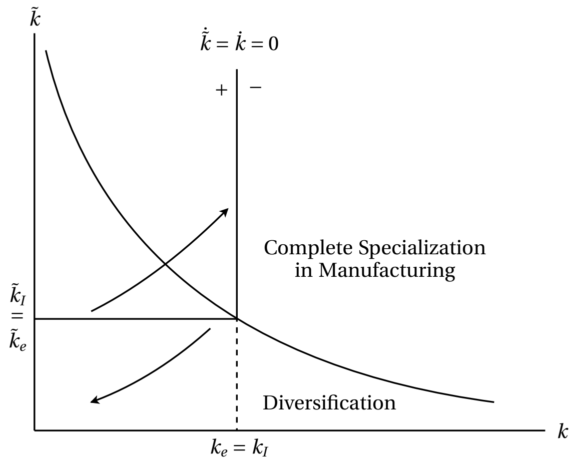

Figure 2. Phase Diagram: $n = n^*$

The Case $n = n^*$ Figure 2 depicts the phase diagram of the home economy for $n = n^*$. Although a diversified steady state exists since $\dot{k} = 0$ and $\dot{\tilde{k}} = 0$ coincide, it is unstable. Given the assumption that $k_0 < k_e$ at the time when free trade is allowed, the economy starts to the left of k_e in Figure 2. Therefore, all depends on whether $\tilde{k}_0 > \tilde{k}_I$ $(= \tilde{k}_e)$ or $\tilde{k}_0 < \tilde{k}_I$, where $\tilde{k}_0$ is the initial value of $\tilde{k}$ at the time when the economy switches from autarky to free trade. In the first case, the home country will eventually specialize in the production of commodity 1, and in the second case, it will asymptotically specialize in the production of commodity 2. As the growth rates at the diversified steady state coincide with the autarkic growth rates (cf. Proposition 2), there is no possibility for the home country to influence its initial position by sticking to autarky for a longer time.

The Case $n < n^*$ As can be seen from Figure 3, the economy asymptotically specializes in agricultural production, regardless of the initial values (cf. Proposition 5). The figure includes just one trajectory that shows how an economy evolves which starts with a comparative advantage in manufacturing (cf. Proposition 3) but has a relatively small rate of population growth. Starting from diversification, the economy specializes in manufacturing for a finite time interval and eventually enters the region of diversification again, from which it converges to asymptotical specialization in agriculture. Thus, an industrialized country with too low a growth rate looses its competitiveness under free trade and eventually becomes deindustrialized. The

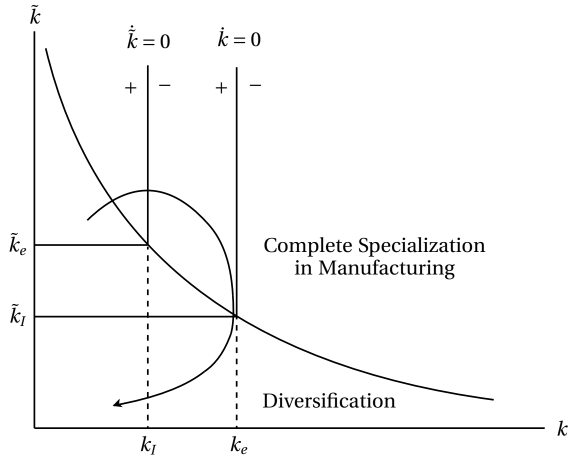

Figure 3. Phase Diagram: $n < n^*$

reason is that the domestic capital stock grows at the rate γn, while in the ROW it grows at the average rate $\gamma n^* > \gamma n$.

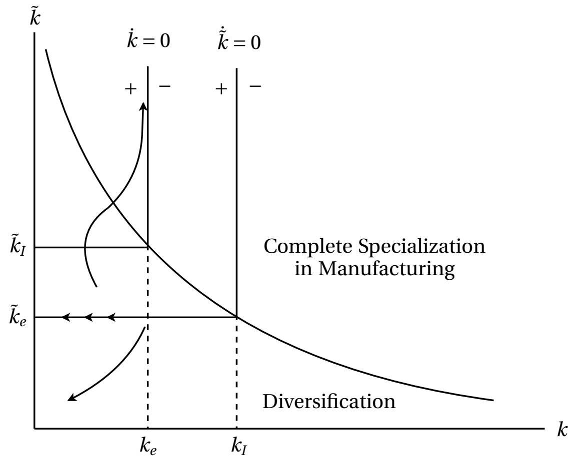

Figure 4. Phase Diagram: $n > n^*$

The Case $n > n^*$ Finally, if $n > n^*$, the situation shown in Figure 4 is similar to the case $n = n^*$. If $\tilde{k}_0 > \tilde{k}_e$, the economy specializes in manufacturing in the long run, and if $\tilde{k}_0 < \tilde{k}_e$, it asymptotically specializes in agriculture.[7] In contrast to the case $n = n^*$, however, the movement of k need not be monotonous. Moreover, as the growth rate g_K of capital under free trade at $\tilde{k} = \tilde{k}_e$ would be γn^* while in autarky it would be $\gamma n > \gamma n^*$, the home country can influence its initial position. If it decides to switch from autarky to free trade later, its capital stock will grow faster than the stock in the ROW. Hence, it is possible to reach an initial value $\tilde{k}_0 > \tilde{k}_e$ and to avoid long-run specialization in agriculture. In this sense, this is the most interesting case.

[7] If $\tilde{k}_0 = \tilde{k}_e$, the economy slides along the isocline $\dot{\tilde{k}} = 0$ with $\lim_{t \to \infty} k(t) = 0$ remaining diversified.

4 Concluding Remarks

The extension of a simple non-scale growth model by a second sector and international trade reveals that the rate of population growth is an important determinant of endogenous comparative advantage, structural change, and long-run growth. International trade leads to a process of industrialization or deindustrialization of the domestic economy, which in turn explains differences in long-run growth rates and in the effects of the population growth on per capita income growth across various countries. E.g., the growth rate of per capita income in terms of manufactured goods increases in the population growth rate in industrialized countries but not in agricultural countries, where the growth rate depends on the growth rate abroad. A country with a relatively high growth rate of population enjoys a positive growth-trade linkage if it has an initial comparative advantage in manufacturing but a negative linkage is possible in case of a disadvantage leading to a process of deindustrialization. A country with a relatively low rate of population growth ($n < n^*$) may even loose an initial comparative advantage in manufacturing and become deindustrialized in the long run. Nevertheless, it gains from international trade because its terms of trade under free trade improve faster than in autarky.

The possibility of deindustrialization in case of a relatively small initial capital stock is akin to the low-level traps known in neoclassical growth theory (for a modern analysis, cf. Deardorff, 2001). The present model goes a step ahead in showing that there are not only low-*level* traps, but also low-*growth rate* traps, which appear in an otherwise well-behaved model of international trade founded on standard assumptions. In fact, it is possible that the long-run per capita growth rate in industrialized countries is higher than in rural countries although the terms of trade of exporters of manufactured goods steadily decline. If agricultural goods were sufficiently inferior to prevent the terms of trade of manufacturing countries from declining, this prediction would be reinforced.

These results show that there is no simple answer to the question about the best development policy (*outward-looking* or *import substitution*). A country with a relatively high growth potential (here measured by the growth rate of population) will suffer from a negative growth-trade linkage related to a process of deindustrialization if it opens up to international trade too early, but it enjoys a higher growth rate under free trade if it is able to start with a comparative advantage in manufacturing. Thus, the timing of the switch from autarky to free trade matters. In this sense, there is no policy ineffectiveness in this non-scale growth model, and some temporary policy measures aimed at protecting infant industries may be reasonable to give structural change a beneficial direction. This prediction accords well with the empirical evidence about the growth of the East Asian newly industrialized countries, e. g. Although these are often quoted as examples for a successful development under free trade, a closer look at the facts reveals that they did not strictly stick to a neutral, outward-looking development strategy (for the case of South-Korea, cf. Pack and Westphal, 1986), but engaged in an industrial policy aiming at structural change towards an accelerated industrialization. Similarly, countries like India and China have liberalized and grown, but a closer look at their timing reveals that their takeoff occurred before trade liberalization (cf. Greenwald and Stiglitz, 2006, p. 145).

While a country with a relatively low growth potential will enjoy a positive growth-trade linkage, it must be noted that this result crucially hinges on the assumed preferences, which imply improving terms of trade of the agricultural societies. Also, measuring the growth potential by the rate of population growth may be appropriate in some cases, but not always. E.g., specific applications require to distinguish between population and the labor force or even human capital and raw labor. A country may also gain from international knowledge spillovers which have been neglected here on the basis of the empirical evidence by Branstetter (2001) as a first approximation to reality. It is obvious that international trade will at least transfer the knowledge about the existence of commodities which are not available at home, however. The flow of such basic knowledge would only be prevented in case of prohibitive protectionism. All of these issues point to possible extensions of the model for future research.

Appendix

A Reduction of the Closed Economy Model to Equation (15)

Substituting (8) into $g_K = \alpha Y_1/K$ [calculated in the main text following (10)] yields

$$g_K = \alpha\theta_1^{1-\alpha} K^{\alpha+\beta-1} L^{1-\alpha} = \alpha\theta_1^{1-\alpha}\left(\frac{K}{L^\gamma}\right)^{\alpha+\beta-1} L^{\gamma(\alpha+\beta-1)} L^{1-\alpha} = \alpha\theta_1^{1-\alpha} k^{\alpha+\beta-1},$$

because, according to the definition of γ, $\gamma(\alpha+\beta-1)+1-\alpha = 0$. From the definition of k, $\dot{k} = g_K k - \gamma n k$, which together with the expression for g_K implies (15). Finally, note that equations (8), (9), and (7) may similarly be expressed in terms of the scale adjusted per capita variables (14):

$$y_1 = \theta_1^{1-\alpha} k^{\alpha+\beta}, \quad y_2 = 1-\theta_1, \quad p = \frac{\theta_1^\alpha}{(1-\alpha)k^{\alpha+\beta}} L^{-\beta/(1-\alpha-\beta)} = \frac{\theta_1^\alpha}{(1-\alpha)k^{\alpha+\beta}} L^{1-\gamma}. \tag{A1}$$

Thus, if k converges to its long-run equilibrium, so do y_1, y_2, and p, where (as $g_L = n$) the latter falls at the constant rate $(1-\gamma)n$ in the steady state.

B Explicit Solution of Equation (20)

Equation (20) is a Bernoulli equation. Thus, it is straightforward to calculate its explicit solution as (cf. any textbook on ordinary differential equations):

$$\tilde{k}(t,\tilde{k}_0) = \left[\left(\tilde{k}_0^{-\beta/\alpha} - \tilde{k}_e^{-\beta/\alpha}\right) e^{\frac{\beta}{\alpha}\gamma n^* t} + \tilde{k}_e^{-\beta/\alpha}\right]^{-\alpha/\beta}. \tag{A2}$$

It is now readily seen that $\lim_{t\to\infty}\tilde{k}(t,\tilde{k}_0) = 0$ if $\tilde{k}_0 < \tilde{k}_e$, and that there exists a $\bar{t} < \infty$ such that $\lim_{t\to\bar{t}}\tilde{k}(t,\tilde{k}_0) = \infty$ if $\tilde{k}_0 > \tilde{k}_e$.

C Proof of Proposition 3

In order to prove Proposition 3, the autarky and world market price ratios must be distinguished. Thus, deviating from notation in the main text, p now denotes the price ratio under autarky at home while p^* is the world market price ratio. Using $k = K/L^\gamma$, a straightforward calculation shows that $L^{1-\gamma} = (K/k)^{-\beta/(1-\alpha)}$. Substituting into (A1) yields

$$p = \frac{\theta_1^\alpha}{(1-\alpha)k^{\alpha/\gamma}} K^{-\beta/(1-\alpha)}$$

Inserting k_e^a from (16) yields the price ratio under autarky for $k_0 = k_e^a$ at time $t = 0$:

$$\text{if} \quad k_0 = k_e^a, \quad \text{then} \quad p = \left(\frac{\gamma n}{\alpha(1-\alpha)^{(1-\alpha)/\alpha}}\right)^{\alpha/(1-\alpha)} K(0)^{-\beta/(1-\alpha)}. \tag{A3}$$

Using the initial value $\tilde{k}_0 = K(0)(p^*)^{(1-\alpha)/\beta}$ of $\tilde{k}$, comparing with $\tilde{k}_e$ provided in (21) and solving for p^* implies

$$\tilde{k}_0 \gtreqless \tilde{k}_e \quad \Longleftrightarrow \quad p^* \gtreqless \left(\frac{\gamma n^*}{\alpha(1-\alpha)^{(1-\alpha)/\alpha}}\right)^{\alpha/(1-\alpha)} K(0)^{-\beta/(1-\alpha)}. \tag{A4}$$

Proposition 3 follows from comparing the price ratios in (A3) and (A4) for the various cases. E.g., if $n < n^*$ and $\tilde{k}_0 \geqq \tilde{k}_e$, the home country has a comparative advantage in manufacturing at $t = 0$:

$$p^* \geqq \left(\frac{\gamma n^*}{\alpha(1-\alpha)^{(1-\alpha)/\alpha}}\right)^{\alpha/(1-\alpha)} K(0)^{-\beta/(1-\alpha)} > \left(\frac{\gamma n}{\alpha(1-\alpha)^{(1-\alpha)/\alpha}}\right)^{\alpha/(1-\alpha)} K(0)^{-\beta/(1-\alpha)} = p.$$

D Asymptotic Specialization in Agriculture

From equation (22), in order to prove asymptotic specialization in agriculture, it has to be shown that $\tilde{k}_0 < \tilde{k}_e$ implies $\lim_{t\to\infty} \tilde{k}(t)^{(\alpha+\beta)/\alpha} p(t)^{-(1-\alpha)/\beta} = 0$. Using equation (A2) and $p(t) = p_0 e^{(1-\gamma)n^* t}$ according to (17), the product on the left hand side in case of $\tilde{k}_0 < \tilde{k}_e$ is

$$p_0^{-(1-\alpha)/\beta} \left[\underbrace{\left(\tilde{k}_0^{-\beta/\alpha} - \tilde{k}_e^{-\beta/\alpha}\right)}_{>0} e^{\beta^2 \gamma n^* t/[\alpha(\alpha+\beta)]} + \tilde{k}_e^{-\beta/\alpha} e^{-\beta\gamma n^* t/(\alpha+\beta)} \right]^{-(\alpha+\beta)/\beta}.$$

It is straightforward that this expression converges to 0 for $t \to \infty$.

E The Isoclines in Figures 2, 3, and 4

The transition line from diversification to specialization in manufacturing in $(k, \tilde{k})$-space can be obtained by setting $Y_2 = 0$ in equation (4) and dividing by L. After some by now well known manipulation, one gets $1 = (1-\alpha) k^{\alpha+\beta} p L^{\beta/(1-\alpha-\beta)}$, where it should be recalled that $k = K/L^\gamma$ according to (14). Substituting $p = \tilde{k}^{\beta/(1-\alpha)} K^{-\beta/(1-\alpha)}$ from the definition of $\tilde{k}$ in (19) and rearranging yields

$$1 = (1-\alpha)^{1-\alpha} k^{\alpha(1-\alpha-\beta)} \tilde{k}^\beta, \tag{A5}$$

the hyperbola in Figures 2, 3, and 4. It is straightforward that below this hyperbola the economy is diversified as long as $(k, \tilde{k}) > (0, 0)$, while above it specializes in manufacturing.

The definitions of k and $\tilde{k}$ in (14) and (19), resp., imply $\tilde{k} = k L^\gamma p^{(1-\alpha)/\beta}$, from which

$$g_{\tilde{k}} = g_k + \gamma(n - n^*). \tag{A6}$$

Substituting $g_{\tilde{k}}$ from (20) yields

$$\dot{k} = \alpha(1-\alpha)^{(1-\alpha)/\alpha} \tilde{k}^{\beta/\alpha} k - \gamma n k, \tag{A7}$$

which together with (20) describes the dynamics in case of diversification. The isoclines $\dot{k} = 0$ and $\dot{\tilde{k}} = 0$ are given by

$$\dot{k} = 0: \qquad \tilde{k} = \tilde{k}_I := \left(\frac{\gamma n}{\alpha(1-\alpha)^{(1-\alpha)/\alpha}}\right)^{\alpha/\beta} \quad (\text{or } k = 0), \tag{A8}$$

$$\dot{\tilde{k}} = 0: \qquad \tilde{k} = \tilde{k}_e := \left(\frac{\gamma n^*}{\alpha(1-\alpha)^{(1-\alpha)/\alpha}}\right)^{\alpha/\beta} \quad (\text{or } \tilde{k} = 0). \tag{A9}$$

In the case of specialization in manufacturing, using (26) together with (A6) yields

$$\dot{\tilde{k}} = \alpha k^{\alpha+\beta-1}\tilde{k} - \gamma n^* \tilde{k}, \tag{A10}$$

which together with (26) describes the dynamics in this case. The isoclines $\dot{k} = 0$ and $\dot{\tilde{k}} = 0$ are given by

$$\dot{k} = 0: \qquad k = k_e := \left(\frac{\alpha}{\gamma n}\right)^{1/(1-\alpha-\beta)} \quad (\text{or } k = 0), \tag{A11}$$

$$\dot{\tilde{k}} = 0: \qquad k = k_I := \left(\frac{\alpha}{\gamma n^*}\right)^{1/(1-\alpha-\beta)} \quad (\text{or } \tilde{k} = 0). \tag{A12}$$

Inserting k_e and $\tilde{k}_I$ or k_I and $\tilde{k}_e$, respectively, into (A5) reveals that the loci $\dot{\tilde{k}} = 0$ and $\dot{k} = 0$ are continuous. The formulas (A8) and (A9) imply that $\dot{\tilde{k}} = 0$ and $\dot{k} = 0$ coincide if $n = n^*$, and that $\dot{\tilde{k}} = 0$ lies above (below) $\dot{k} = 0$ if $n < n^*$ $(n > n^*)$.

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