Journal of Economics manuscript No.
(will be inserted by the editor)

On the Implications of Declining Population Growth for Regional Migration

Thomas Christiaans

This is the accepted version of the article originally submitted to Journal of Economics. The final publication is available at link.springer.com.

Abstract Although many advanced economies nowadays experience decreasing populations, migration in models of economic growth has so far been almost exclusively analyzed for the case of non-negative population growth rates. This paper considers decreasing and possibly negative population growth rates in two two-sector growth models. As long as preferences are homothetic, neither a decrease in population growth rates nor an actual population decline does induce migration in either direction. Introducing quasi-linear preferences implies that a decline in population growth leads to migration from the rural to the industrial region. A complete depopulation of the rural region takes place if the population growth rate falls short of minus the rate of physical capital depreciation. These results reinforce pessimistic expectations about a rural exodus.

Keywords (Negative) Population Growth · Migration · Two-Sector Growth Model · Quasi-Linear Preferences
JEL-Classification J61 · O41 · R11

1 Introduction

The theories of economic growth and migration are mostly concerned with non-negative population growth rates. Following a long history of increasing populations, however, many industrialized countries nowadays experience a decline of their populations or at least have growth rates of population near zero. Especially in Europe only four countries have experienced above-replacement fertility during any 5-year period since 1990–1995; European population is prospected to decline from 738 millions in

Thomas Christiaans
FOM University of Applied Sciences
E-mail: thomas.christiaans@fom.de

2015 to 707 millions in 2050 (United Nations, Department of Economic and Social Affairs, Population Division, 2015, pp. 1 and 10). While there are a few theoretical papers on economic growth that consider the implications of negative population growth (Samuelson, 1975; Ritschl, 1985; Ferrara, 2011; Christiaans, 2011; Sasaki and Hoshida, 2016, e.g.), to the best of my knowledge there is just one theoretical analysis of migration in the context of economic growth with declining populations (Christiaans, 2012). Nevertheless, some economists have very pessimistic expectations about the future consequences of shrinking populations, especially with respect to the development of rural regions. The following quotation provides a clear example:

> ”With the strong contraction of population the younger people will start to move to the congested urban areas, both from the country and from the smaller urban areas. The elderly will stay as long as possible, but will finally have to follow. The material infrastructure in the abandoned areas will decay. The institutions: enterprises, public administration, police, schools, universities, hospitals will disappear from these areas. They will not gently separate, so that the last to go switches off the light, but they will break down as suddenly as in an earthquake and leave chaos as after an earthquake. The material values invested in the province will be lost. E.g., houses become worthless, even if they are not already ruined, because nobody wants to buy them anymore.
> An atmosphere of gloom and doom will spread as on a sinking ship, and it will expand to the metropolises. Law and order cannot be upheld and the civil society will break down. Contractual claims and therefore also claims on insurances can only be established by private force, the economy will continue to shrink more than it would correspond to the contraction of the population. The elderly and the sick will be the first to suffer from the evolving barbarity. Social security will break down before the collapse of the entire society.” (Brinkmann, 2009, pp. 11–12, translation by author)

While this quotation in some respects is akin to a prophecy, it nevertheless comprises serious arguments suitable to raise fears about the future development of shrinking societies shared by many researchers.[1] According to Reher (2007, p. 189), e.g., the decline in population will be the ”key social issue of the twenty-first century”. An interesting question that has not been answered as yet, however, is whether standard neoclassical theory would also strengthen or lighten such fears. It is shown in Christiaans (2011) for the case of a semi-endogenous growth model that there is indeed a case for pessimistic expectations about the growth performance of a country experiencing a decline of its population. For a wide range of reasonable parameter values, a negative population growth rate induces a decline in per-capita income.[2] The purpose of the present paper is to investigate whether, in addition to assertions about the macroeconomic growth performance, a prediction about a rural exodus can also be substantiated by a neoclassical model of interregional migration in a growth context.

As an actual example, a study of the German Bundesinstitut für Bau-, Stadt- und Raumforschung establishes a renaissance of cities since 2000. All but one of the ten fastest growing German cities over the period 2000–2009 have more than 500.000 inhabitants (Göddecke-Stellmann, 2011, p. 5). The smaller towns and villages in the rural regions suffered a decline of their populations since 2004 (Göddecke-Stellmann, 2011, p. 9). During 2007–2013, the mean annual growth rates of population in urban

---

[1] As a matter of course, there are also less pessimistic views, cf. Gáková and Dijkstra (2010), e.g.

[2] Although Sasaki and Hoshida (2016) arrive at more optimistic conclusions, it should be noted that these are attributable to the fact that the rate of capital depreciation is zero in their model, giving rise to an advantageous capital deepening effect.

districts has been 0.48% in western and 0.89% in eastern Germany, while in several varieties of rural districts the rates have all been negative and ranged from −0.08% to −1.28% (Milbert, 2015, p. 6). Remarkably the actual development therefore matches Brinkmann's prophecy originally stated in January 2001 as part of his valedictory lecture. While it is understood that a monocausal explanation does not do justice to the phenomenon of migration to the cities, it is an important issue whether standard neoclassical arguments do support or moderate the thesis of a rural exodus.

Countries like Germany considered in the preceding paragraph are not the classical examples used to explain notions like "rural exodus" or "urbanization". It is therefore remarkable that even in Germany as in other European countries such a renaissance of cities can be observed in present days. Worldwide the process of urbanization is even more pronounced, as the following figures illustrate (cf. United Nations, Department of Economic and Social Affairs, Population Division, 2014, p. 7): In 1950, about 70 per cent of world population lived in rural areas. In 2014, about 54 per cent of the world's population live in urban regions. For the first time in history, the global urban population exceeded the global rural population in 2007. The United Nation's projection for 2050 is that about 66 per cent of population will live in urban areas.

Modern neoclassical growth theory can roughly be classified into the theory of endogenous and the theory of semi-endogenous growth (cf. Christiaans, 2004, e.g.). Most of the models of endogenous growth involve scale effects in that a larger population leads to a higher growth rate of per-capita income. Therefore, a steady state does not exist whenever the population growth rate does not vanish. Thus, introducing migration in such a model means that overall population must either be constant as e.g. in Glomm (1992), Premer and Walz (1994) or Lucas (2004), or population growth can only be introduced as a one-time change in the size of population as e.g. in the North-South model of Lundborg and Segerstrom (2002). Moreover, endogenous growth is always a knife-edge case. Models of endogenous growth are therefore not suitable to analyze migration in case of a negative population growth rate. Nevertheless, the following analysis is related to Glomm (1992) in that the influence of quasi-linear preferences will finally be analyzed. While Glomm (1992) concludes that there will be a permanent migration from the rural to the urban region (a complete rural exodus), the main conclusion of the present paper is that such an entire exodus will take place only if the overall population growth rate falls short of minus the rate of depreciation of physical capital. Nevertheless, a decline in the population growth rate will always induce some additional migration into the urban region.

The present paper uses a standard two-sector growth and migration model as a starting point and sticks to the theory of semi-endogenous growth by considering variations of this model. To keep matters as simple as possible, it concentrates on a model without long-run per-capita growth. This simplicity allows for an analytical solution of a model with non-homothetic preferences that is able to at least partly explain migration from rural to urban areas as a result of a decline in population growth. A related model by Reichlin and Rustichini (1998) concentrates on diverging growth patterns in a two-country setting, each producing the same good, in an overlapping generations model. In contrast, the present paper is concerned with two regions producing different goods and simplifies the analysis by neglecting overlapping generations. The main focus of the analysis is to identify forces that determine the impact of negative population growth on the directions of migration in such a setting.

The analysis starts in Sec. 2.1 with the standard neoclassical two-sector growth and migration model of Mas-Colell and Razin (1973) and investigates the implications of declining population growth rates that eventually become negative. The result is that these changes do not induce migration in either direction. Sec. 2.2 starts with a brief review of the model of semi-endogenous growth and migration analyzed in Christiaans (2012), where again the decline in population does not induce any migration. The main reason for this behavior turns out to be the assumption of homothetic preferences in both of the cases. Therefore, a similar model is presented that simplifies matters by assuming technical progress away but introduces quasi-linear preferences. A decline in population growth now leads to migration from the rural to the industrial region, thereby reinforcing pessimistic expectations about a rural exodus. The final section concludes and argues that there is indeed a case for pessimism, because the neutrality of population growth with respect to migration in the other models is based on an unrealistic assumption about consumer preferences.

2 Two Simple Models of Rural-Urban Migration

2.1 The Neoclassical Two-Sector Growth Model

Mas-Colell and Razin (1973) considered intersectoral migration in the standard neoclassical two-sector growth model first rigorously analyzed by Uzawa (1961, 1963). This model has also been interpreted as a model of interregional migration (cf. e.g. Barro and Sala-i-Martin, 2003). As is standard in growth theory, the population growth rate was assumed to be a positive constant without considering the implications of different population growth rates. This section briefly summarizes the model of Mas-Colell and Razin (1973) and extends their analysis by investigating the effects of a declining and eventually negative population growth rate on migration.

There are two regions described by their respective production functions, both of which are of the Cobb-Douglas type, $Y_i = K_i^{\alpha_i} L_i^{1-\alpha_i}$, $i = 1, 2$, where K_i and L_i are the capital and the labor input, respectively (notation differs from Mas-Colell and Razin, 1973). As capital is instantaneously transferred between regions, competition equalizes the marginal productivities of capital (in terms of the numéraire, good 2) in both regions. The allocation of labor, however, does not adjust instantaneously to wage rate differentials. Therefore, wages in both regions may differ. The industrial wage rate is denoted w_1 and the rural wage rate is w_2, both measured in agricultural goods per unit of labor.

The agricultural good 2 serves as the numéraire and p is the price of the industrial good 1 in terms of the agricultural good. While the agricultural sector produces a pure consumption good, the industrial good can be consumed or invested. Per capita national income in terms of agricultural goods is $py_1 + y_2$, $y_i = Y_i/L$, where L is population, which for simplicity shall equal the labor force. It is assumed that a constant proportion $s > 0$ of income is spent for saving and also that a constant proportion

$\zeta > 0$ of income is spent on the industrial good for consumption, implying that the underlying preferences must be homothetic. Both rates together must be smaller than one: $0 < s + \zeta < 1$. Let $\lambda = s/(s+\zeta)$ and $\xi = \alpha_1 s/[\alpha_1 s + \alpha_2(\lambda - s)]$ and assume that population grows at an exogenous and constant rate n, that is $g_L := \dot{L}/L = n$. (In general, derivatives with respect to time are indicated by a dot and the growth rate of any variable x is denoted as g_x.) Then Mas-Colell and Razin (1973) show that the development of the aggregate capital-labor ratio $k = K/L$ is governed by the differential equation

$$\dot{k} = \lambda \xi^{\alpha_1} k^{\alpha_1} \rho^{1-\alpha_1} - nk, \tag{1}$$

where $\rho = L_1/L$. Beginning with the case $n > 0$, the isocline $\dot{k} = 0$ in (k, ρ)-space is

$$\rho = \left(\frac{n}{\lambda \xi^{\alpha_1}}\right)^{1/(1-\alpha_1)} k \tag{2}$$

This is a straight line through the origin. Since $d\dot{k}/d\rho > 0$ from (1), $\dot{k} > 0$ above this line, while $\dot{k} < 0$ below this line. This is indicated by the $+$ and $-$ signs next to $\dot{k} = 0$ in Fig. 1.

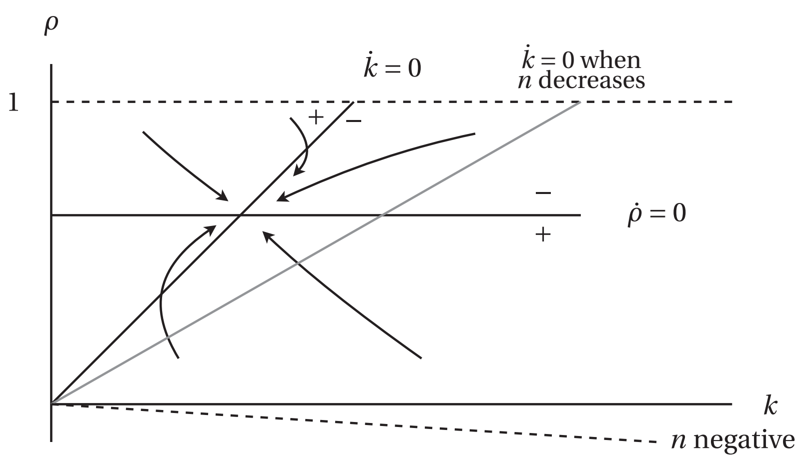

Fig. 1 Phase diagram of the Mas-Colell-Razin model for various population growth rates

Migration depends on the difference in regional wage rates. Let M be migration into the industrialized region, then $M + nL_1$ is the change $\dot{L}_1$ of population in this region. Thus, as $g_\rho = g_{L_1} - g_L$, the growth rate of ρ equals the rate of migration into the industrialized region:

$$\frac{\dot{\rho}}{\rho} = \frac{\dot{L}_1}{L_1} - \frac{\dot{L}}{L} = \frac{M + nL_1}{L_1} - n = \frac{M}{L_1} \tag{3}$$

Let $\mu > 0$. Then migration is assumed to follow the equation

$$\frac{\dot{\rho}}{\rho} = \mu \left[\frac{w_1 - w_2}{w_2}\right]$$

Calculating the marginal productivities of labor in both regions Mas-Colell and Razin (1973) show that this assumption leads to the following equation determining the development of labor allocation:

$$\frac{\dot{\rho}}{\rho} = \mu \left[\frac{1-\alpha_1}{\alpha_1} \frac{\alpha_2}{1-\alpha_2} \frac{1-\rho}{\rho} - 1 \right] \tag{4}$$

Setting (4) equal to zero yields the $\dot{\rho} = 0$ isocline

$$\rho = \frac{(1-\alpha_1)\alpha_2}{(1-\alpha_1)\alpha_2 + (1-\alpha_2)\alpha_1},$$

shown in Fig. 1. As this isocline is a horizontal line, it determines the steady state value of ρ and therefore the long-run rates of labor located in the two regions independently of the aggregate capital-labor ratio k. Since $d\dot{\rho}/d\rho < 0$ from (4), $\dot{\rho} < 0$ above the isocline and $\dot{\rho} > 0$ below the isocline. This is indicated by the $+$ and $-$ signs next to $\dot{\rho} = 0$ in Fig. 1.

The phase diagram reveals that there is a unique and globally stable long-run equilibrium (cf. the representative trajectories in Fig. 1) for all positive initial values of k and ρ. Regarding the issue raised in the present paper, a particular feature of this model is that a change in the rate of population growth, n, will not lead to migration in either direction. This result follows from the fact that a change in n does only change the slope of the isocline $\dot{k} = 0$ while leaving the horizontal isocline $\dot{\rho} = 0$ unaffected. E.g., if the rate of population growth decreases, the isocline $\dot{k} = 0$ will rotate clockwise giving rise to an new long-run equilibrium entailing a higher capital-labor ratio and an unaltered fraction of labor in region one, ρ (cf. the gray line in Fig. 1). If the economy starts at the old long-run equilibrium when the population growth rate goes down, even the transitional dynamics will not involve any migration since the movement will occur along the isocline $\dot{\rho} = 0$.

Notice that even if n becomes negative, the long-run equilibrium will disappear but ρ stays at the same steady-state level as before, although k increases without bound, cf. the dashed line in Fig. 1. It is understood that the region of negative values for ρ is of course not feasible because, first, ρ must be non-negative, and second, even considering negative values of ρ would imply that (2) is not the solution of (1) for all admissible values of α_1. The location of this hypothetical (and thus dashed) isocline nevertheless clearly visualizes that $\dot{k} > 0$ regardless of the value of $\rho \in [0,1]$. At this point it also becomes apparent that neglecting the depreciation of capital in case of a decreasing population is not an assumption as innocent as in the case of an increasing population. Considering a positive rate δ of capital depreciation would imply the existence of a steady state even for $n < 0$ as long as $n + \delta > 0$. Cf. Christiaans (2011) for the case of a one-sector growth model and also the final paragraph in Sec. 2.2.

Considering the neutrality of population growth with respect to migration the question arises whether this is a robust result or a result depending particularly on any special assumption of the model. Apart from the standard assumption that markets are perfectly competitive the central assumptions are first that the production functions are Cobb-Douglas, second that there is no technical progress, and third that preferences with respect to consumption of the two goods are homothetic. Considering the first assumption, Mas-Colell and Razin (1973, p. 75) point to the fact that the equilibrium level of ρ, the fraction of labor located in the industrial region, will depend on the aggregate capital-labor ratio k if the production functions are not Cobb-Douglas. As k in turn depends on the population growth rate n, migration would therefore not be independent of population growth in such a generalized model as analyzed by Bosch et al (1973). The exact influence of n on migration would be very difficult to analyze, however, and clear-cut results regarding the direction of the impact could not be derived since they would depend in a complicated manner on elasticities of substitution and other parameters of the model. Moreover, a generalization along the lines of the modern theory of endogenous or semi-endogenous growth would not allow for much more general production functions than Cobb-Douglas (cf. Eicher and Turnovsky, 1999, p. 404).

The following subsection thus considers the second and third assumption. The latter turns out to be crucial with respect to the effects of population growth on migration.

2.2 Linear Agricultural Production and Quasi-Linear Preferences

A somewhat simplified Uzawa-type model with migration is analyzed in Christiaans (2012). It builds on ideas set forth in Wong and Yip (1999), who consider a scale model of endogenous growth that has been reformulated as a non-scale semi-endogenous growth model in Christiaans (2008) and Sasaki (2011), e.g. As a model of semi-endogenous growth, it involves a variation of the second main assumption mentioned before by introducing technical progress. The simplicity of this model stems from using an agricultural production function linear in labor as a limiting case of a Cobb-Douglas function where the labor elasticity is one while the capital elasticity is zero. One of the results in Christiaans (2012) is that in the long run migration is independent of population growth, just as in Mas-Colell and Razin (1973).

This result can swiftly be proven without explicitly considering the dynamics of migration, because the fractions of labor located in both regions in long-run equilibrium are independent of population growth. While the agricultural production function is linear in labor, there are increasing returns to scale in the industrial sector using capital and labor due to learning externalities. The production functions are

$$Y_1 = K^{\alpha+\beta} L_1^{1-\alpha}, \quad Y_2 = L_2,$$

where K is capital exclusively used in region 1 and L_i, $i = 1, 2$, is labor input in region i. The parameter β measures the extent of learning-by-doing externalities. Region 1 produces industrial goods while region 2 produces agricultural goods. While agricultural goods are just consumed, the industrial goods can be consumed or invested. Thus, capital K is measured in the same units as Y_1. The agricultural good serves as the numéraire and p denotes the price of the industrial good in terms of the agricultural good.

Households determine their consumption according to the classical saving function, that is, all capital income is saved while all labor income is consumed.[3] The aggregate consumption quantities C_1 and C_2 are determined by maximizing a Cobb-Douglas utility function $U(C_1, C_2) = C_1^{\theta} C_2^{1-\theta}$, $0 < \theta < 1$, subject to the budget constraint. In a migration equilibrium, wage rates in terms of the agricultural good in both regions must be equalized and equal to one, since the marginal productivity of labor in region 2 is one. Thus, in long-run equilibrium the budget constraint reads $pC_1 + C_2 = L$. Consumption demand is therefore

$$pC_1 = \theta L \quad \text{and} \quad C_2 = (1-\theta)L.$$

Equilibrium on the market for agriculture requires $Y_2 = C_2$, that is

$$L_2 = (1-\theta)L, \quad \text{or} \quad L_2/L = 1-\theta.$$

Using the full employment condition, $L_1 + L_2 = L$, and denoting the fraction of labor in industrial production as ρ implies $L_2/L = 1-\rho$. Combining with the preceding equation yields

$$\rho = \theta.$$

This result implies that in every migration equilibrium in this model the fraction of labor in industry is a constant determined by the preference parameter θ and that it is independent of the population growth rate n. The model thus shares the implication obtained in Sec. 2.1 that a change in population growth will not induce any migration in either direction. Notice that preferences in both models are homothetic.

The third assumption about preferences thus appears to be crucial with respect to the effects of population growth on migration. In order to concentrate on the effects of population growth on migration in case of non-homothetic preferences, matters are simplified by disregarding technical progress. It will turn out that a decrease of the population growth rate will induce migration into the industrialized region in that case. Thus, the production functions now read

$$Y_1 = K^{\alpha} L_1^{1-\alpha}, \quad Y_2 = L_2.$$

Further assumptions are as before. Using the full employment condition $L = L_1 + L_2$, the production functions in per capita terms read:

$$y_1 = k^{\alpha} \rho^{1-\alpha}, \tag{5}$$

$$y_2 = 1-\rho, \tag{6}$$

where $y_1 = Y_1/L$, $k = K/L$, $y_2 = Y_2/L$, and $\rho = L_1/L$, $0 \leqq \rho \leqq 1$.

The agricultural good serves as the numéraire. Let p denote the price of the industrial good in terms of the agricultural good. As capital is just needed in industrial production, the marginal productivity of capital in terms of the numéraire is

$$p\alpha k^{\alpha-1} \rho^{1-\alpha}$$

[3] Compared to a fixed saving rate, the classical saving function involves some degree of rationality since in its extreme version it implies convergence to the golden rule path in the absence of externalities (cf. Burmeister, 1980, p. 60). It may therefore be interpreted as a simple rule of thumb for consumers out of their depth with dynamic optimization methods.

Competition within the industrial region therefore implies that the net rental rate of capital in terms of the industrial good 1 is

$$r = \alpha k^{\alpha-1}\rho^{1-\alpha} - \delta, \tag{7}$$

where δ is the the rate of capital depreciation.

The allocation of labor does not adjust instantaneously. Therefore, wages in both regions may differ (both measured in agricultural goods per unit of labor). Denoting the industrial wage rate by w_1 and the rural wage rate by w_2, competition within the regions implies that wages equal the respective marginal productivities of labor:

$$w_1 = p(1-\alpha)k^{\alpha}\rho^{-\alpha}, \quad w_2 = 1. \tag{8}$$

As before it is assumed that all capital income is saved while all labor income is consumed (the classical saving function). A short-run equilibrium where saving equals net investment shall always prevail. Then, using (7), net investment $\dot{K}$ and the growth rate of capital g_K must respectively equal

$$\dot{K} = rK = (\alpha k^{\alpha-1}\rho^{1-\alpha} - \delta)K \quad \text{and} \quad g_K = \alpha k^{\alpha-1}\rho^{1-\alpha} - \delta$$

(In general, derivatives with respect to time are indicated by a dot and the growth rate of any variable x is denoted as g_x.) Using $k = K/L$, $\dot{L}/L = n$ and $g_k = g_K - n$ implies

$$\dot{k} = \alpha k^{\alpha}\rho^{1-\alpha} - (\delta + n)k \tag{9}$$

Beginning with the case $\delta + n > 0$, the isocline $\dot{k} = 0$ in (k, ρ)-space is

$$\rho = \left(\frac{\delta + n}{\alpha}\right)^{1/(1-\alpha)} k \tag{10}$$

This is a straight line through the origin defined for $0 \leqq \rho \leqq 1$. Since $d\dot{k}/d\rho > 0$ from (9) for $0 < \rho < 1$, $\dot{k} > 0$ above the isocline and $\dot{k} < 0$ below the isocline. This is indicated by the $+$ and $-$ signs next to $\dot{k} = 0$ in Fig. 2.

The standard approach to non-homothetic preferences found in the literature is modeling preferences by a Stone-Geary utility function with subsistence consumption (cf. e.g. Matsuyama, 1992; Steger, 2000; Strulik, 2010). Although this assumption helps in explaining real world phenomena that are out of reach with homothetic preferences (e.g. the link between agricultural productivity and the growth rate of an economy or the increase in saving rates during economic development), they also have two shortcomings. First, models of economic growth become more difficult to analyze, especially in case of two sectors, and second, it unrealistically implies that with increasing income and constant prices agricultural consumption increases linearly. Following Glomm (1992), as an alternative approach it will be assumed that preferences can be described by a quasi-linear utility function, implying that agricultural goods are consumed first (depending on prices) while additional income is spent on industrial goods.

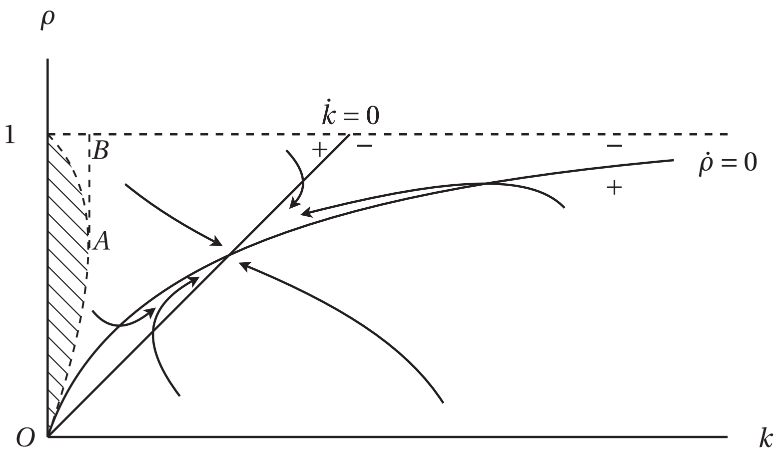

Fig. 2 Phase diagram for $\delta+n>0$

Consumption demand is determined using a quasi-linear utility function u for each household,

$$u = c_1 + c_2^{\theta}, \quad 0<\theta<1,$$

where c_i denotes per capita consumption of good i. The quasi-linear utility function is a special case of the preferences analyzed by Gorman (1953) and therefore allows for a simple aggregation procedure leading to well-behaved aggregate demand functions provided that all households consume positive quantities of both goods. It is shown in App. A that per capita consumption of the agricultural good is

$$c_2 = (\theta p)^{1/(1-\theta)},$$

because income distribution is always such that all households can buy the stated amount of the agricultural good to the right of the shaded area in Fig. 2. As long as k and ρ do not enter this area, aggregate demand for agriculture is therefore $(\theta p)^{1/(1-\theta)}L$. Equating this demanded quantity to its supply, $L-L_1$, gives the equilibrium price of the industrial good 1 in terms of the agricultural good 2:

$$p = \frac{1}{\theta}(1-\rho)^{1-\theta} \tag{11}$$

Migration shall again depend on the relative difference in regional wage rates. As shown in (3), the growth rate of ρ equals the rate of migration into the industrialized region. As before, population is assumed to grow at the rate n. Let $\mu>0$. Then, using (8) and (11), migration follows the equation

$$\dot{\rho} = \mu\rho\left[\frac{w_1-w_2}{w_2}\right] = \mu\left[\frac{1}{\theta}(1-\rho)^{1-\theta}(1-\alpha)k^{\alpha}\rho^{1-\alpha}-\rho\right] \tag{12}$$

Setting (12) equal to zero yields the $\dot{\rho}=0$ isocline

$$k = \left(\frac{\theta}{1-\alpha}\frac{\rho^{\alpha}}{(1-\rho)^{1-\theta}}\right)^{1/\alpha} \tag{13}$$

It is shown in App. B that the first and the second derivative of this isocline with respect to ρ are both positive and that $k = 0$ if $\rho = 0$, and $k \to \infty$ if $\rho \to 1$. Thus, the isocline has the shape shown in Fig. 2 (notice that axes are interchanged). It is also shown in App. A that this isocline always lies to the right of the shaded area in Fig. 2 (except at the origin). As $d\dot{\rho}/dk > 0$ for $0 < \rho < 1$ from (12), $\dot{\rho} < 0$ to the left of the isocline and $\dot{\rho} > 0$ to the right of the isocline, indicated by the $+$ and $-$ signs next to the isocline $\dot{\rho} = 0$.

Whether a long-run equilibrium with positive values of k and ρ exists depends on the slopes of the isoclines $\dot{k} = 0$ and $\dot{\rho} = 0$ at the origin. As shown in App. B, the slope of (13) at the origin is $[(1-\alpha)/\theta]^{1/\alpha}$. An equilibrium therefore exists if the slope of (10) is smaller:

$$\left(\frac{\delta + n}{\alpha}\right)^{1/(1-\alpha)} < \left(\frac{1-\alpha}{\theta}\right)^{1/\alpha}.$$

Although it is not possible to prove that this inequality is generally met, it will be valid for all reasonable parameter values because α and θ, being elasticities of the production and utility functions, respectively, should be an order of magnitude larger than δ and n, the rate of capital depreciation and the rate of population growth. E.g., if $\alpha = 0.5$, $\theta = 0.5$, $\delta = 0.05$, and $n = 0.01$, the above inequality reads $0.12^2 < 1^2$. It will therefore be assumed in the sequel that the isocline $\dot{\rho} = 0$ is steeper than $\dot{k} = 0$ at the origin. In this case both zero isoclines intersect and a positive long-run equilibrium exists.

In order to assure that the shaded area where the consumption functions would change will never be entered one further qualification is necessary. At the margin of the shaded area in Fig. 2, trajectories point to the south-east. Along the dashed curve from point O to point A, all trajectories point out of that region. This is not generally assured along the curve between points A and $(0,1)$. Fig. 2 therefore includes an auxiliary line from A to B. Each trajectory starting on the curve OAB points south-east and will never enter the shaded area. The same applies to the following figures. As the representative trajectories in Fig. 2 indicate, the positive equilibrium is globally stable provided that the initial point is not located to the left of the curve OAB and both k and ρ are positive.

The analysis of the influence of the population growth rate n on the long-run equilibrium and thus on migration and capital accumulation is now straightforward. As a first step, suppose that n decreases but that $n + \delta$ remains positive. As can be seen from (10), the isocline $\dot{k} = 0$ becomes flatter, as illustrated in Fig. 3, while the isocline $\dot{\rho} = 0$ does not change since (13) is independent of n. The new, stable equilibrium will therefore involve higher values of k and ρ. In other words, the long-run aggregate capital intensity increases and a larger part of population moves to the industrialized region.

The intuition behind this result is as follows. The decrease in the rate of population growth leads to an increase in the growth of the per-capita capital stock and therefore an increasing per-capita capital stock itself, cf. Eq. (9). This implies according to (8) that the marginal productivity of labor in industry increases, while in

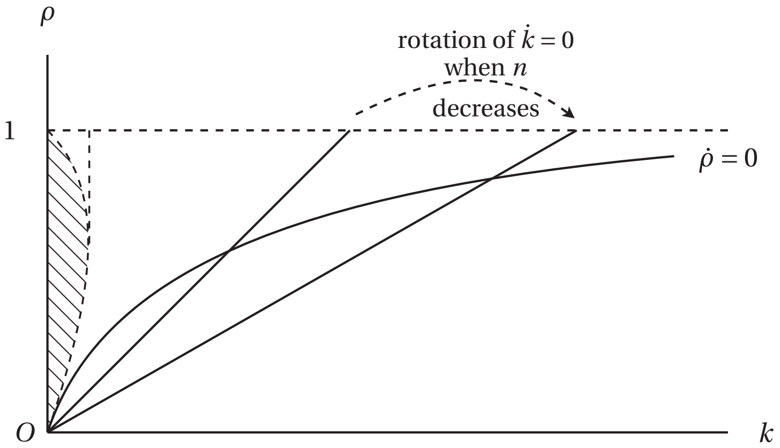

Fig. 3 Shift of equilibrium when n decreases

agriculture it stays constant.[4] In the standard models with homothetic preferences, the resulting increase in industrial production would lead to a deterioration of the relative price of industrial goods that completely offsets the increase in labor productivity, leaving the industrial wage rate in terms of agriculture unchanged. Given the quasi-linear preferences used in the present model, however, the relative price of industrial goods does not deteriorate initially and therefore the increase in industrial labor productivity induces a migration of workers from the rural to the industrial region, cf. Eq. (12). As can be seen from (11), the price of industrial goods deteriorates only as a secondary effect of the inflow of workers in industrial production that leads to an increase in ρ.

Now suppose that $n+\delta$ becomes negative. Fig. 4 shows the resulting phase diagram with a dashed isocline $\dot{k} = 0$ in order to emphasize again that the region of negative values for ρ is of course not feasible (cf. the discussion of the dashed line in Fig. 1 on p. 6). It illustrates, however, that $\dot{k}$ is positive for all $k > 0$ regardless of the value of $\rho \in [0,1]$. The phase diagram therefore implies that $k \to \infty$ and $\rho \to 1$ as $t \to \infty$. In other words, the rural region becomes depopulated before the whole economy breaks down (due to extinction of its population).

The case $\delta + n < 0$ serves to emphasize another important implication of the analysis. While neglecting the depreciation of capital in models of economic growth and migration with a positive growth rate of population generally appears to be a harmless simplification of the analysis, it would be an oversimplification in case of negative population growth. If $\delta = 0$, Fig. 4 would apply for any negative population growth rate. If $\delta > 0$, however, Fig. 3 would still apply as long as $\delta > -n$, which appears to be the empirically more relevant case. As an example, if population was -1% per year, $\delta > -n$ would be met as long as δ was larger than 1%, which should

[4] It should be noted that, as long as industrial production is more capital intensive, this intuition will also apply in the more general case where capital is used as an input in agriculture, too. Although the marginal productivity of labor in agriculture would not be constant in that case, it would not increase as much as in industry.

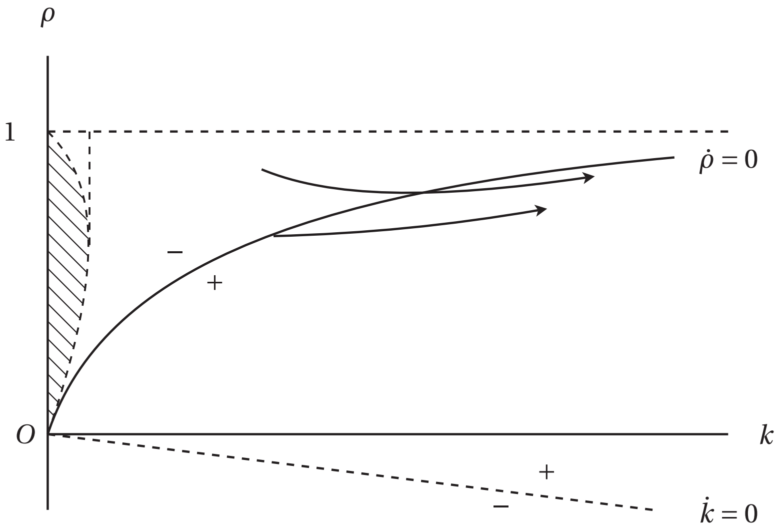

Fig. 4 Phase diagram for $\delta + n < 0$

generally be true since otherwise the half-value period of capital goods would be 69 ($\approx \ln(0.5)/-0.01$) years ore more. Referring to a one-sector growth model, it is shown in Christiaans (2011) that neglecting depreciation in case of negative population growth rates leads to erroneous predictions about long-run per capita growth.

3 Conclusions

The paper started with a pessimistic prophecy about the implications of a decreasing population on migration to urban areas leading to a rural exodus. Many of the arguments that are used to substantiate such a prophecy, e.g. the preference for an urban lifestyle, are out of the scope of neoclassical models of growth and migration. It has been shown that if preferences with respect to industrial and agricultural goods are not homothetic and better described by quasi-linear preferences, the neoclassical theory nevertheless reinforces this pessimism. A decrease in the rate of population growth induces migration out of the rural areas. Although quasi-linear preferences obviously are a simplification of reality, they appear to be more realistic than homothetic or even Stone-Geary preferences that would both imply a constant marginal propensity to consume with respect to agricultural goods and therefore a linear increase of agricultural consumption in income.

There are two theoretical by-products of the analysis. First, it has been shown that a two-sector model of growth and migration can be solved analytically even in case of quasi-linear preferences. The ease of the analysis stems from the fact that it is possible to define regions in the phase diagrams where all households consume positive quantities of both goods leading to simple aggregate demand functions. Second, the analysis shows that neglecting depreciation, while often being a harmless assumption in case of positive population growth, is not as harmless when population decreases. Neglecting depreciation leads to long-run dynamics completely different from the more realistic case of a rate of depreciation that is larger than minus the negative population growth rate.

The simplicity of the model should also prove useful when trying to extend the model. An important issue that may be raised in future research is the analysis of optimum saving decisions in the presence of negative population growth rates and migration from a normative point of view. It should also be possible to analyze a similar model that includes semi-endogenous growth. As has been shown in Christiaans (2011) for a one-sector model of semi-endogenous growth, negative population growth implies negative per-capita income growth for a wide range of reasonable parameter values. As this reinforces the pessimism about the future development of a shrinking society, the analysis of optimum saving decisions and optimum policy measures to control migration flows becomes even more important. From an economic policy point of view the most important result is that one argument has been added to emphasize the urgency for economic policy measures that prevent a further population decline.

Appendix

A Consumption Demand

According to the classical saving function, all wage income is consumed. There are two types of wage earners, rural workers and industrial workers. From (8), a rural worker receives the wage rate $w_2 = 1$ and therefore maximizes his utility $u = c_1 + c_2^\theta$ subject to $pc_1 + c_2 = 1$. The result of this optimization neglecting non-negativity constraints is $c_2 = (\theta p)^{1/(1-\theta)}$ for agricultural goods. It will now be shown that no non-negativity constraint is binding outside the shaded areas in Figures 2–4 and therefore the derived demand function is valid. Substituting the demand for good 2 and (11) into the budget equation yields

$$c_1 = \theta\rho(1-\rho)^{\theta-1} > 0 \quad \text{for} \quad 0 < \rho < 1,$$

showing that no non-negativity constraint becomes active for rural workers.

Workers in the industrialized region receive the wage rate $w_1 = p(1-\alpha)k^\alpha\rho^{-\alpha}$ according to (8). Substituting $c_2 = (\theta p)^{1/(1-\theta)}$ and (11) into $pc_1 + c_2 = p(1-\alpha)k^\alpha\rho^{-\alpha}$ yields

$$c_1 = (1-\alpha)k^\alpha\rho^{-\alpha} - \theta(1-\rho)^\theta,$$

which is non-negative if k lies to the right of the locus defined by

$$k = \left(\frac{\theta}{1-\alpha}(1-\rho)^\theta\rho^\alpha\right)^{1/\alpha}. \tag{A1}$$

Comparing this expression with (13) shows that (A1) defines a curve that coincides with (13) only if $\rho = 0$, is located to the left of (13) if $0 < \rho \leqq 1$, and has the shape of the dashed curve in Fig. 2, cf. App. B. In other words, for any $\rho \in (0,1]$, k in (13) exceeds k in (A1). Thus, to the right of the locus defined by (A1), that is, outside of the shaded area in Fig. 2, and especially along the isocline $\dot{\rho} = 0$, consumption of the industrial good by industrial workers is positive and no non-negativity constraint becomes active. Therefore, the per-capita consumption of good 2 of every worker indeed is $c_2 = (\theta p)^{1/(1-\theta)}$ and aggregate consumption is $(\theta p)^{1/(1-\theta)}L$. An explicit formula giving the aggregate consumption of good 1 could be derived but is not needed for the analysis.

B The Isocline (13) and the Curve (A1)

Using the abbreviation $b = [\theta/(1-\alpha)]^{1/\alpha} > 0$, the isocline $\dot{\rho} = 0$ given in (13) is rewritten as

$$k = \frac{b\rho}{(1-\rho)^{(1-\theta)/\alpha}}.$$

Notice at first that $k = 0$ if $\rho = 0$, and $k \to \infty$ as $\rho \to 1$. The derivative of k with respect to ρ is

$$\frac{dk}{d\rho} = b\left[(1-\rho)^{-(1-\theta)/\alpha} + \frac{1-\theta}{\alpha}\rho(1-\rho)^{-(1-\theta+\alpha)/\alpha}\right] > 0.$$

Next, the slope at the origin is

$$\lim_{\rho\to 0}\left(\frac{dk}{d\rho}\right) = b = \left(\frac{\theta}{1-\alpha}\right)^{1/\alpha}.$$

As axes are interchanged in Fig. 2 and the slope of the isocline at the origin is important as to whether a positive equilibrium exists, notice that $\lim_{k\to 0}(d\rho/dk) = [(1-\alpha)/\theta]^{1/\alpha}$.

The second derivative is

$$\frac{d^2k}{d\rho^2} = b\left[2\frac{1-\theta}{\alpha}(1-\rho)^{-(1-\theta+\alpha)/\alpha} + \frac{1-\theta}{\alpha}\frac{1-\theta+\alpha}{\alpha}\rho(1-\rho)^{-(1-\theta+2\alpha)/\alpha}\right] > 0,$$

proving the shape of the isocline shown in Fig. 2.

Next, using again the abbreviation $b = [\theta/(1-\alpha)]^{1/\alpha}$, the slope of (A1) is

$$\frac{dk}{d\rho} = b\left[(1-\rho)^{\theta/\alpha} - \frac{\theta}{\alpha}\rho(1-\rho)^{(\theta-\alpha)/\alpha}\right]$$

Notice that, as with (13), $\lim_{\rho\to 0}(dk/d\rho) = b$.

As $k = 0$ if $\rho = 0$, and $k = 0$ if $\rho = 1$, the curve must have a maximum between 0 and 1. The first derivative vanishes only if $\rho = \alpha/(\theta+\alpha)$, which thus is the maximizing value of ρ. (A1) has therefore approximately the shape shown in Fig. 2.

References

Barro RJ, Sala-i-Martin X (2003) Economic Growth, 2nd edn. MIT Press, Cambridge

Bosch A, Mas-Colell A, Razin A (1973) Instantaneous and non-instantaneous adjustment to equilibrium in two-sector growth models. Metroeconomica 25:105–118

Brinkmann G (2009) Die Zukunft der deutschen Sozialversicherung, Diskussionspapier Nr. 139-09, Universität Siegen

Burmeister E (1980) Capital Theory and Dynamics. Cambridge University Press, Cambridge

Christiaans T (2004) Types of balanced growth. Economics Letters 82:253–258

Christiaans T (2008) International trade and industrialization in a non-scale model of economic growth. Structural Change and Economic Dynamics 19:221–236

Christiaans T (2011) Semi-endogenous growth when population is decreasing. Economics Bulletin 31:2667–2673

Christiaans T (2012) A non-scale growth model of migration and structural change. In: Lv C, Zhang S, Fan Y, Zhang H (eds) Conference Proceedings on Transformation of Resource-Based Economy and Internationalization of Higher Education, Aussino, Riverwood, pp 266–275

Eicher TS, Turnovsky SJ (1999) Non-scale models of economic growth. Economic Journal 109:394–415
Ferrara M (2011) An AK Solow-Swan model with a non-positive rate of population growth. Applied Mathematical Sciences 5:1241–1244
Gáková Z, Dijkstra L (2010) Does population decline lead to economic decline in EU rural regions? Regional focus – Eurpean Union – Regional policy, 01/2010, Brussel
Glomm G (1992) A model of growth and migration. The Canadian Journal of Economics 25:901–922
Göddecke-Stellmann J (2011) Renaissance der Großstädte – eine Zwischenbilanz. BBSR-Berichte KOMPAKT 9/2011, Bundesinstitut für Bau-, Stadt- und Raumforschung, Bonn
Gorman WM (1953) Community preference fields. Econometrica 21:63–80
Lucas RE Jr (2004) Life earnings and rural–urban migration. Journal of Political Economy 112:S29–S59
Lundborg P, Segerstrom PS (2002) The growth and welfare effects of international mass migration. Journal of International Economics 56:177–204
Mas-Colell A, Razin A (1973) A model of intersectoral migration and growth. Oxford Economic Papers 25:72–79
Matsuyama K (1992) Agricultural productivity, comparative advantage, and economic growth. Journal of Economic Theory 58:317–334
Milbert A (2015) Wachsen oder schrumpfen? BBSR-Analysen Kompakt 12/2015, Bundesinstitut für Bau-, Stadt- und Raumforschung, Bonn
Premer M, Walz U (1994) Divergent regional development, factor mobility, and nontraded goods. Regional Science and Urban Economics 24:707–722
Reher DS (2007) Towards long-term population decline: a discussion of relevant issues. European Journal of Population / Revue européenne de Démographie 23:189–207
Reichlin P, Rustichini A (1998) Diverging patterns with endogenous labor migration. Journal of Economic Dynamics and Control 22:703–728
Ritschl A (1985) On the stability of the steady-state when population is decreasing. Journal of Economics 45:161–170
Samuelson PA (1975) The optimum growth rate for population. International Economic Review 16:531–538
Sasaki H (2011) Trade, non-scale growth and uneven development. Metroeconomica 62:691–711
Sasaki H, Hoshida K (2016) The effects of negative population growth: An analysis using a semiendogenous R&D growth model. Macroeconomic Dynamics, First View:1–16
Steger TM (2000) Economic growth with subsistence consumption. Journal of Development Economics 62:343–361
Strulik H (2010) A note on economic growth with subsistence consumption. Macroeconomic Dynamics 14:763–771
United Nations, Department of Economic and Social Affairs, Population Division (2014) World urbanization prospects: The 2014 revision, highlights st/esa/ser.a/352

United Nations, Department of Economic and Social Affairs, Population Division (2015) World population prospects: The 2015 revision, key findings and advance tables. working paper no. esa/p/wp.241

Uzawa H (1961) On a two-sector model of economic growth. Review of Economic Studies 29:40–47

Uzawa H (1963) On a two-sector model of economic growth ii. Review of Economic Studies 30:105–118

Wong KY, Yip CK (1999) Industrialization, economic growth, and international trade. In: Choi EK, Jensen BS (eds) Economic Growth and International Trade, Blackwell, Oxford, pp 164–182