Regional Competition for the Location of New Facilities

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Abstract. A model of interregional competition for the location of new facilities is analyzed as a differential game. Two regions try to enhance their attraction by making concessions to a location decision maker in order to raise the probability that a new facility will be located in a specific region. The prospective benefits and costs of exerting influence are decisive for the final outcomes of the model. The open-loop (and feedback) Nash equilibrium solution is inefficient in comparison to the cooperative solution of joint benefit maximization of both regions.

JEL-Classification: R38; C73

1 Introduction

Interregional competition for the location of new (production) facilities by a location decision maker (LDM) has recently been analyzed in Jutila (2001). Two (or more) regions try to enhance their attraction by making concessions to the LDM, defining *attraction* as the probability that the LDM will locate his facility in a specific region. The benefit of having a new facility located in a region consists of the number of new jobs, new income etc. As Jutila (2001) remarks, it is "rather obvious that regions are competing for jobs and income in a rather dynamically changing environment". However, he describes this dynamical game rather mechanically, without explicitly considering the objective functions of the regions.

The present paper suggests a model of the competition for location decisions between two regions as an explicit differential game.[1] Since the actions of one region in this dynamical setting directly influence the attraction of the other region and since these actions are generally costly and should therefore be set off against the prospective benefits of having a new facility located in the region, this is the typical setting of a differential game.

It should be emphasized that the framework of the present model is entirely different from the one usually employed in location games. According to Thisse (1987, p. 519), "the primary purpose of location theory is to explain the spatial distribution

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[1]In comparison with Jutila (2001), however, we simplify the model in other respects: We consider only two regions and we neglect the direct influences of the LDM on the ongoing competition process. Our dynamical system describing the development of the probabilities is different from Jutila's.

of production activities in an economy". This explanation is attempted by considering the optimum location in space from the viewpoint of competing firms. The theory of location games applies game theoretic concepts to this end.[2] In contrast, we are considering the game in a dynamical setting from the viewpoint of two competing regions that try to influence an LDM who has announced that he has almost completed his decision process and indicates that his final decision at some future date T depends on the concessions made by both of the regions and their governments, respectively. Thus, we ignore the primary optimization process of the firm and of its LDM with respect to transportation cost minimization etc., and assume that the LDM has already ascertained two almost equally good alternatives (with respect to transportation costs etc.). The last stage of his optimization process then consists in encouraging the two regions considered to make as many concessions as possible. We analyze the actions of these regions to influence the LDM's final decision between the two possible locations.

The empirical relevance of this setting is emphasized by Jutila (2001, p. 59), who remarks that "regions resort to intensified promotional, marketing and public relations activities in order to create a positive attractive image to LDMs." As an example, he gives a detailed description of the profile that Northwest Ohio, U.S.A., uses as a marketing device in order to raise its attraction to LDMs. Moreover, his paper includes a case study of a plant location decision. In Europe, the discussion about the harmonization of tax rates provides ample evidence for the importance of attracting (foreign) direct investment to regions and countries. Black and Hoyt (1989) and Haaparanta (1996) give actual examples of very high subsidies paid by governments to firms in order to attract investment of new facilities.

The framework just described bears some resemblance to the theory of tax competition. Like location games, tax competition models have their origins in the urban economics and regional science literature, e.g. cf. Beck (1983). In these models, regional or national governments display Nash behavior and strive to maximize their objectives, which may be regional or national product or simply tax revenue (the Leviathan assumption). These objectives are pursued by choosing appropriate rates of capital (income) taxation in order to attract capital, which is assumed to be interregionally (internationally) mobile and perfectly divisible.[3] While these approaches can naturally address some questions that are outside the scope of the present model, they are generally static and written in a macroeconomic spirit with homogeneous capital and no special reference to a particular plant location decision.

In contrast, we analyze the competition of two regions for the location of a new facility in the dynamic framework of a differential game. Regions do not only have capital taxes or subsidies at their disposal. They use various promotional activities and do also compete in the supply of local public goods (material infrastructure), cf. e.g. Jutila (2001). These instruments involve costs that come about during the process of competition for the location of new facilities; hence, they require a dynamic analysis. The formulation of our model is general enough to capture such dynamic measures. While the present model does not focus on the investigation of taxes and subsidies as instruments of regional competition, these instruments can be taken into account indirectly (cf. footnote 10). To the best of my knowledge, no such model is available

[2]For a review of the theory of location games, cf. Gabszewicz and Thisse (1992).

[3]A recent (critical) review of tax competition models can be found in Koch and Schulze (1998).

up to now, although the two articles of Black and Hoyt (1989) and Haaparanta (1996) model interregional or international competition for the location of firms. These authors also employ static equilibrium approaches and discuss some issues that are outside the scope of the present paper, which abstracts from some details in order to take a step into the direction of an appropriate dynamic analysis that sheds light on other issues than capital taxation and subsidies.

The analysis is not confined to private location problems; it can also be applied to the location of public facilities. The activities of the regions to enhance their attraction then comprise political manipulation at the local, national and international level. Even competition for the location of major global events, such as the World Cup or the Olympic Games, falls within the realm of the model. For example, countries competing for the organization of the Olympic Games invest in material infrastructure and try to influence members of the IOC during the period from the competitors' application to the final vote.[4]

Section 2 describes the basic framework of the analysis. A simple example that admits an explicit solution of the model is provided in section 3, whereas some more general results are derived in section 4. The efficiency of the outcomes is then analyzed in section 5 by comparing the Nash equilibrium solution with the cooperative solution of joint benefit maximization for both regions. We will finally discuss some possible extensions of the model.

2 The Model

We consider two regions, $R1$ and $R2$, and a location decision maker (LDM) who decides to locate a new facility in one of the two regions. The game starts at time $t = 0$ and the decision is being made at time $t = T$. The flow of the monetary benefit of having the new facility located in region i, $i = 1,2$, is b_i at every point in time (and zero otherwise). Thus, b_i is measured in monetary units and its value reflects the monetary advantages provided by the location of a new facility (e.g. new jobs or new income).[5] The probability of having the facility located in region i from time T on is $p_i(T)$. In the Nash equilibrium to be considered below, $p_i(T)$ is equal to its expectation at times $0 \leq t < T$. Thus, at times $0 \leq t < T$, the expectation of the flow $U_i(t)$ of monetary benefit can be written as

$$U_i(t) = \begin{cases} 0 & : \ 0 \leq t < T \\ p_i(T)b_i & : \ T \leq t < \infty. \end{cases} \tag{1}$$

The probability $p_i(T)$ that the LDM decides to locate the facility in Ri can be influenced by the regions according to the following differential equations, where a dot

[4]Since the final decision is accomplished by a vote of many IOC members, the analysis of this last stage of the game belongs to the theory of social choice. Cf. Eichner et al. (1996).

[5]That b_i is assumed to be constant simplifies the exposition but barely restricts the generality of the model. As can be seen from equation (6) below, the bequest-value $p_i(T)b_i/\rho$ would be replaced by a more complicated but still constant expression if b_i was a function of time.

denotes the derivative with respect to time:[6]

$$\begin{aligned} \dot{p}_1 &= A - p_1, \\ \dot{p}_2 &= B - p_2. \end{aligned} \tag{2}$$

Here, $B := 1 - A$, and $A = A(u_1, u_2)$ is a differentiable function of the control variables u_1 and u_2 with

$$\begin{gathered} A_1 := \frac{\partial A}{\partial u_1} > 0, \quad A_{11} < 0, \quad A_2 := \frac{\partial A}{\partial u_2} < 0, \quad A_{22} > 0, \\ A(u_1, u_2) \in [0,1] \quad \forall u_1 \geq 0,\ u_2 \geq 0, \\ \text{and} \quad A(n,m) = 1 - A(m,n) \quad \forall n \geq 0,\ m \geq 0. \end{gathered} \tag{3}$$

The control variable u_1 is a policy instrument used by region 1 in order to shift attraction from region 2 to region 1, and u_2 has an analogous meaning. The policy variable u_1 can be interpreted as a collection of all possible actions that $R1$ can use in order to influence the LDM. Since these actions comprise a broad range of measures, it is most natural to interpret u_1 as a kind of a Hicksian composite good, that is, an index number constructed by summing over the various instruments weighted with some base prices. This assumption does not preclude that one of the regions, say $R2$, needs to invest more actual dollars than the other, $R1$, in order to provide a given amount of this composite good. E.g., $R2$ could face higher administrative costs, or the LDM may simply require more concessions from $R2$ than from $R1$ in order to generate the same effect on A.

Conditions (3) have the following meaning: $R1$ ($R2$) can raise (reduce) A and thereby increase its attraction with diminishing returns.[7] The function A is defined for all nonnegative values of u_1, u_2 and can take on values between 0 and 1, which implies that the probabilities cannot escape the same range. Finally, the last assumption in (3) implies that $A(n,n) = 1/2\ \forall n \geq 0$, so that the long-run probabilities for $t \to \infty$ are equal to $1/2$ for both regions if they choose the same value of u_i. Due to this symmetry assumption, the possibility of influencing the LDM is equal in both regions. This is a natural assumption since possible differences in the endeavor of exerting influence are taken into account by differences in the cost functions of both regions that are considered below.

The probabilities $p_i(T)$ are the values of the variables $p_i(t)$ at time T. Clearly, these variables should satisfy

$$p_i(t) \geq 0, \qquad p_1(t) + p_2(t) = 1, \quad \text{and} \quad \dot{p}_2 = -\dot{p}_1 \quad \forall t,$$

which, since $B := 1 - A$ has been assumed, is easily seen to be true if the initial values $p_1(0) = p_{10}$ and $p_2(0) = p_{20}$ satisfy the constraints[8]

$$p_{20} = 1 - p_{10} \geq 0, \quad p_{10} \geq 0. \tag{4}$$

[6] A similar specification has been used by Asada (1997) in order to model the number of trips using the transportation services of two firms. Without a condition such as (3), however, his assumptions do not appear to be sufficient for keeping the state variable in its domain of definition.

[7] If A exhibited changeable returns to u_1 but the Hamiltonian H_1 in (7) for all t attained its maximum at points where A is locally concave, the analysis below would still apply. If, however, A was globally convex in u_1, we would have to deal with solutions of the bang-bang type (assuming an upper bound for the admissible values of u_1).

[8] Note that $\dot{p}_1 + \dot{p}_2 = A - p_1 + 1 - A - p_2 = 1 - p_1 - p_2 = 0\ \forall t$ if $p_{20} = 1 - p_{10}$.

Therefore, the second equation in (2) can be written as

$$\dot{p}_2 = (1-A) - (1-p_1) = p_1 - A;$$

this equation is redundant and can be neglected in the sequel. Thus, it suffices to consider

$$\dot{p}_1 = A(u_1, u_2) - p_1. \tag{5}$$

We are now in a position to explain more thoroughly the economic interpretation of equations (2), respectively (5). Suppose that $p_{10} = 0.5$ and $u_1 = u_2$ at some initial time $t = 0$, such that $\dot{p}_1(0) = 0$. If one of the regions, say $R1$, now decides to invest more effort and raises u_1 while u_2 remains constant, it can raise its attraction because A increases from 0.5 to a higher value. However, this process of gaining (as well as losing) attraction requires time, and this is exactly what is captured by (5). While there are of course other functional forms that could be used to model this kind of behavior, (5) is a reasonable and especially simple differential equation that exhibits the desired property. At the same time, (5) satisfies the requirement that the numerical values of $p_1(t)$, a probability, must lie between zero and one.

As has been noted before, we assume that the regions possibly need to raise different amounts of money in order to provide the same quantity of the composite good u_i. This is captured by cost functions $C_i(u_i)$ for exerting influence, which are assumed to be convex and to involve no fixed costs. For simplicity, we set $C_i(u_i) = c_i u_i$, where $c_i > 0$ is the constant per unit cost of exerting influence in Ri. This assumption barely restricts the generality of the model because we have already assumed diminishing returns with respect to the function A.[9] Therefore, the flow of the monetary net benefit at time t is $U_i(t) - C_i(u_i(t))$. We assume that both of the regions are risk neutral, which implies that the integral of the discounted flow of the expected monetary net benefit from time 0 to ∞ is a reasonable objective function to maximize. Let ρ with $0 < \rho < 1$ be the common discount rate for both regions. Using equation (1) and the fact that $u_i(t) = 0$ and therefore $c_i u_i(t) = 0$ is obviously optimal from time T on,[10] the expected cumulated monetary net benefit of Ri is

$$J_i = \int_0^\infty [U_i(t) - C_i(u_i(t))] e^{-\rho t}\, dt = e^{-\rho T} p_i(T) b_i/\rho - \int_0^T c_i u_i(t) e^{-\rho t}\, dt, \tag{6}$$

which has to be maximized given equations (5) and (4). Thus, the problem of maximizing over an infinite time interval has been reduced to a finite-time problem with a discounted bequest-value

$$e^{-\rho T} S_i(p_i(T)) := e^{-\rho T} p_i(T) b_i/\rho.$$

We analyze the problem with strategies in *open-loop*, which for the present case means that $R1$ ($R2$ resp.) maximizes its objective function with respect to $u_1(t)$ ($u_2(t)$

[9] For example, the main effect of a strictly convex cost function C_1 would be to make the Hamiltonian H_1 in (7) strictly concave in u_1; however, this is already the case because of diminishing returns of A with respect to u_1.

[10] If the winning region promised payments to the LDM for the time succeeding T, e.g. a constant subsidy per period, the model would still apply. In this case, b_i has to be interpreted as monetary benefit net of subsidies. To compensate for a higher subsidy of one of the regions, c_i could have a lower value for this region.

resp.) given the time-path of $u_2(t)$ ($u_1(t)$ resp.) without *feedback* control. The open-loop Nash equilibrium will be reached if both regions correctly anticipate the time-path of their respective competitor, each of which is optimal in the indicated sense. No region can put itself at an advantage by unilaterally deviating from the Nash equilibrium strategy to another open-loop strategy. The respective problems of each of the regions can be solved using Pontryagin's maximum principle.

Due to its simple structure, the model is state-separable, that is, the determination of the controls and the costate variables is separated from the determination of the state variables. This implies in turn that the open-loop Nash equilibrium, if it exists, is also a degenerate (because of independence of the current state) feedback Nash equilibrium that does not depend on the initial state and is therefore subgame perfect.[11] Moreover, as will be shown in section 4, there is only one open-loop equilibrium. Thus, there is no need to *refine* further the open-loop Nash equilibrium, which is already subgame perfect and unique.

The current value Hamiltonians for regions 1 and 2 are

$$\begin{aligned} H_1 &= -c_1 u_1 + \lambda_1 [A(u_1, u_2) - p_1], \\ H_2 &= -c_2 u_2 + \lambda_2 [A(u_1, u_2) - p_1]. \end{aligned} \tag{7}$$

As with (7), henceforth the first equation concerns $R1$ and the second concerns $R2$. While the respective equations for the individual regions describe their relevant optimization problems, the simultaneous solution of all equations together yields the Nash equilibrium of the game.

The necessary equilibrium conditions with respect to the control variables u_1 and u_2 include

$$\begin{aligned} \frac{\partial H_1}{\partial u_1} &= -c_1 + \lambda_1 \frac{\partial A}{\partial u_1} = 0, \\ \frac{\partial H_2}{\partial u_2} &= -c_2 + \lambda_2 \frac{\partial A}{\partial u_2} = 0, \end{aligned} \tag{8}$$

where for the moment we assume an interior solution for simplicity. Note that the assumptions (3) together with the convexity of the cost functions imply that the equations (8) determine the unique maxima of the Hamiltonians with respect to the controls u_1 and u_2 respectively, because it is seen from (9) and (10) below that λ_1 is positive while λ_2 is negative. The costate variables λ_i must satisfy

$$\begin{aligned} \dot{\lambda}_1 &= \rho \lambda_1 - \frac{\partial H_1}{\partial p_1} = \rho \lambda_1 + \lambda_1, \\ \dot{\lambda}_2 &= \rho \lambda_2 - \frac{\partial H_2}{\partial p_1} = \rho \lambda_2 + \lambda_2. \end{aligned} \tag{9}$$

Finally, the transversality conditions are

$$\begin{aligned} \lambda_1(T) &= \frac{\partial S_1}{\partial p_1(T)} = b_1/\rho, \\ \lambda_2(T) &= \frac{\partial S_2}{\partial p_1(T)} = -b_2/\rho. \end{aligned} \tag{10}$$

[11] See Fershtman (1987). For the concept of state-separability cf. Dockner et al. (1985). A comprehensive account of the theory of non-cooperative differential games as well as a short introduction to Pontryagin's maximum principle can be found in Başar and Olsder (1995).

Given that $\lambda_1 > 0$ for all $t \in [0,T]$ and $\lambda_2 < 0$ for all $t \in [0,T]$, it is easily shown that the Hamiltonians H_1 respectively H_2 are concave in (u_1, p_1) respectively (u_2, p_1). Since $S_1(p_1(T))$ and $S_2(1-p_1(T))$ are concave in $p_1(T)$, this implies that the necessary conditions (8), (9), and (10) are also sufficient conditions for a Nash equilibrium.

The equations (8), (9), and (10) together with (5) and (4) can be reduced to a system of three differential equations with one initial and two transversality conditions, either in p_1, λ_1 and λ_2, or in p_1, u_1 and u_2. The solution of this boundary-value problem yields the Nash equilibrium trajectories $u_i(t)$ of the game. We start with the solution of a simple example in the next section and then return to the more general case.

3 A Specific Example

In order to derive an explicit solution of the game, we use a concrete version of the function $A(u_1,u_2)$. A reasonable and simple candidate that satisfies the assumptions (3) is

$$A(u_1,u_2) = \frac{1}{2} - \frac{1}{2(1+u_1)} + \frac{1}{2(1+u_2)}.$$

This specification given, the solutions of the equations (8) with respect to u_i are

$$\begin{aligned} u_1 &= \sqrt{\lambda_1/(2c_1)} - 1, \\ u_2 &= \sqrt{-\lambda_2/(2c_2)} - 1. \end{aligned} \tag{11}$$

From (9) and (10), the solutions of the linear differential equations for $\lambda_i(t)$ are easily calculated to be

$$\begin{aligned} \lambda_1(t) &= \frac{b_1}{\rho} e^{(1+\rho)(t-T)}, \\ \lambda_2(t) &= -\frac{b_2}{\rho} e^{(1+\rho)(t-T)}. \end{aligned} \tag{12}$$

Substituting (12) into (11) now yields the open-loop Nash equilibrium trajectories

$$\begin{aligned} u_1(t) &= \sqrt{b_1/(2c_1\rho)} e^{(1+\rho)(t-T)/2} - 1, \\ u_2(t) &= \sqrt{b_2/(2c_2\rho)} e^{(1+\rho)(t-T)/2} - 1, \end{aligned} \tag{13}$$

which, as mentioned before, are independent of the initial state and therefore are subgame perfect degenerate feedback strategies. It should be noted that, for the sake of notational simplicity, we do not use extra symbols for the optimum strategies and denote them simply as $u_i(t)$.

The equations (13) are only valid if the non-negativity conditions are not violated. However, the assumption of interior solutions seems reasonable because b_i/ρ, the present value of the new facility in *Ri* calculated at time T, should be a much greater number than c_i in order to have a reasonable problem. Therefore, if the planning horizon T is not too large, both of the u_i are positive for all $t \in [0,T]$. A necessary but not sufficient condition for an interior solution is $b_i/(2c_i\rho) > 1$. On the other hand, if T is large enough, the non-negativity conditions may be effective at the beginning of the game even if $b_i/(2c_i\rho)$ is much greater than one. In this case, as can be seen from (11), the equilibrium strategies are $u_i(t) = 0$ during a period lasting from $t = 0$ to some $T_i \in (0, T)$ defined by $\lambda_i(T_i) = 2c_i$. From T_i on, the strategy of Ri is given by (13).

We neglect the case of effective non-negativity constraints in the following because it involves only minor variations of the main arguments. Equation (5) now reads

$$\dot{p}_1 = \frac{1}{2} - \frac{1}{2(1+u_1)} + \frac{1}{2(1+u_2)} - p_1, \quad p_1(0) = p_{10}.$$

Substitution of (13) into this equation yields the non-autonomous linear differential equation for p_1:

$$\dot{p}_1 = -p_1 + \frac{1}{2} + Ke^{-(1+\rho)(t-T)/2},$$

where K is the constant

$$K = \frac{\sqrt{b_1/(2c_1\rho)} - \sqrt{b_2/(2c_2\rho)}}{2\sqrt{b_1/(2c_1\rho)}\sqrt{b_2/(2c_2\rho)}}.$$

The general solution of the homogeneous part of the equation is Ce^{-t}, where C is an arbitrary constant, and a particular solution of the nonhomogeneous equation can be found using the variation of the constant formula. The solution of the initial value problem turns out to be

$$p_1(t) = \left(p_{10} - \frac{1}{2} - \frac{2K}{1-\rho}e^{(1+\rho)T/2}\right)e^{-t} + \frac{1}{2} + \frac{2K}{1-\rho}e^{-(1+\rho)(t-T)/2}. \tag{14}$$

The most important result concerning $p_1(t)$ is its value at time T. From (14),

$$p_1(T) = \frac{1}{2} + \left(p_{10} - \frac{1}{2}\right)e^{-T} + \frac{2K}{1-\rho}\left(1 - e^{(\rho-1)T/2}\right), \tag{15}$$

and it should be recalled that $p_2(T) = 1 - p_1(T)$. Note also that $1 - e^{(\rho-1)T/2} > 0$ because $0 < \rho < 1$.

The main conclusions of this example are drawn from considering the equations (13) and (15) and are summarized as follows:

1. From (13), the optimum subgame perfect policy functions $u_i(t)$ of both regions in an open-loop Nash equilibrium are strictly monotonously increasing in time t (except for a possible initial interval of inactivity). As one would have suspected, $u_i(t)$ at every given point in time rises with the benefit b_i and falls with the per unit cost of exerting influence c_i and the discount rate ρ. At every point in time is $u_1(t) > u_2(t)$ if and only if $b_1/c_1 > b_2/c_2$.[12]

2. From (15), the probability of having the new facility located in $R1$ is greater than the probability of a location in $R2$ if and only if the sum of the last two terms is

[12]It is important to notice that these results, especially the result concerning the monotonous increase of $u_i(t)$ in t, do not depend on the discount rate ρ being positive. For $\rho = 0$, the problem has to be modeled slightly differently in order to have a convergent objective function. For example, it could be assumed that the benefit b_i is only positive until a certain point in time $\bar{T} > T$. Other things being equal, the results of such a specification with $\rho = 0$ are similar to the results obtained so far for $\rho > 0$.

positive. For example, if the LDM is indifferent between both regions at time 0 (i.e. $p_{10} = p_{20} = 1/2$), it depends on the value of K which region is more likely to be preferred at time T. Clearly, $K = 0$ if both regions are identical. If the regions are not identical, however,

$$K \gtreqless 0 \quad \text{iff} \quad b_1/c_1 \gtreqless b_2/c_2,$$

that is, whether $p_1(T)$ or $p_2(T)$ is greater depends on the ratios of the flows of the respective benefit to the respective per unit cost of exerting influence.

3. The preceding discussion has not necessarily determined that the probability $p_i(T)$ is higher for the region with a higher flow of benefit b_i, even if $p_{10} = p_{20}$, because this effect can easily be outweighed by the cost effect. If, for example, $b_1 < b_2$ but c_1 is sufficiently smaller than c_2, $p_1(T)$ can be greater than $p_2(T)$. Thus, if a region has lower per unit costs of exerting influence – e.g. due to a closer familiarity with the LDM – it may be more likely preferred with a relatively low net benefit. Therefore, the LDM's decision may be inefficient from a social point of view.

4 Generalization

We return to the more general case with an unspecified function $A(u_1, u_2)$ satisfying conditions (3). We are going to investigate whether the three main conclusions drawn from the specific example in the last section continue to be valid or not. The results may be summarized as follows. The first conclusion can only be verified with a qualification, but it nevertheless appears to be valid for a broad range of functions A. Although it cannot be proved that the controls $u_i(t)$ are both strictly monotonously increasing in t, at least one of them is, and as long as the absolute value of the cross partial A_{12} is not too high, both will be increasing. The conclusion that $u_1(t) > u_2(t)$ if and only if $b_1/c_1 > b_2/c_2$ is valid in general without restrictions. The same applies to the validity of the second and the third conclusion, which can definitely be answered in the affirmative.

While it is naturally impossible to get explicit solutions for the strategies now, the solutions for the costate variables are given by (12) as before. If we substitute (12) into equations (8), then we get a system of two equations describing implicitly the evolution of the $u_i(t)$:

$$\begin{aligned} A_1(u_1, u_2) &= \frac{c_1 \rho}{b_1} e^{(1+\rho)(T-t)}, \\ A_2(u_1, u_2) &= -\frac{c_2 \rho}{b_2} e^{(1+\rho)(T-t)}. \end{aligned} \tag{16}$$

At any given point in time, u_1 and u_2 can be viewed as given by (16) as functions of c_i and b_i, $i = 1, 2$. Now, define the function $\phi(t) := \rho e^{(1+\rho)(T-t)}$ and the parameters $a_i := b_i/c_i$, $i = 1, 2$, and differentiate (16) with respect to u_i and a_i to get

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} du_1 \\ du_2 \end{pmatrix} = \begin{pmatrix} -(\phi(t)/a_1^2) da_1 \\ (\phi(t)/a_2^2) da_2 \end{pmatrix}, \tag{17}$$

where the matrix on the left-hand side is abbreviated as $\mathbf{A}$. From assumptions (3) and if A is twice continuously differentiable ($A \in C^2$), it follows that $|\mathbf{A}| = A_{11}A_{22} - A_{12}A_{21} < 0$, because $A_{11} < 0 < A_{22}$ and $A_{12} = A_{21}$ for all $u_1 \geq 0$, $u_2 \geq 0$. Thus, all principal minors of the Jacobian do not vanish for $u_1 \geq 0, u_2 \geq 0$, which, by a well known theorem of Gale and Nikaidô (1965, p. 91), implies the global univalence of the mapping on the left-hand side of (16). Hence, assuming enough variation of the first order derivatives of the function A, this system of equations has a globally unique solution for $u_1(t)$ and $u_2(t)$. (If one or both of the non-negativity constraints are effective, (16) has no positive solution. This case will be neglected in the sequel.) Since the costates are uniquely determined by (12) and (16) is a necessary equilibrium condition, the global univalence implies the uniqueness of the open-loop Nash equilibrium. Since $\mathbf{A}$ is invertible, the solution of the matrix equation (17) is

$$\begin{pmatrix} du_1 \\ du_2 \end{pmatrix} = \frac{1}{|\mathbf{A}|} \begin{pmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{pmatrix} \begin{pmatrix} -(\phi(t)/a_1^2)da_1 \\ (\phi(t)/a_2^2)da_2 \end{pmatrix}. \tag{18}$$

To evaluate the signs of du_i, we need some more information on the function A that can be obtained from (3). The condition $A(n,m) = 1 - A(m,n)$ implies $A_1(n,m) = -A_2(m,n)$ and $A_{12}(n,m) = -A_{21}(m,n)$. If $A \in C^2$, $A_{12}(u_1,u_2) = A_{21}(u_1,u_2)$. Therefore, if $u_1 = u_2 = n$, the last two equations imply $A_{12}(n,n) = 0$. Next, observe that $a_1 = a_2$ implies $u_1 = u_2$.[13] Thus, $A_{12}(n,n) = A_{21}(n,n) = 0$ if $a_1 = a_2$. Hence, starting from a symmetric situation with $a_1 = a_2$ and $da_1 > 0 = da_2$, (18) implies

$$\frac{\partial u_1}{\partial a_1} = -\frac{\phi(t)}{a_1^2}\frac{A_{22}}{|\mathbf{A}|} > 0, \quad \frac{\partial u_2}{\partial a_1} = 0. \tag{19}$$

While this is only a local result at a first glance, a deeper investigation shows that it establishes $u_1(t) > u_2(t)$ for all $t \in [0,T]$ if $a_1 > a_2$, because both functions u_i are continuous in a_i and (19) shows that u_1 rises – starting at a symmetric situation – with a_1 above u_2. If $u_1 = u_2$ at a later point in time (or for a value of a_1 that has been rising again), this would imply $a_1 = a_2$ from (3) and (16), which contradicts the assumption $a_1 > a_2$.

The next step is to investigate the dependence of $u_i(t)$ on time t. Differentiating (16) with respect to t yields, similar to (18),

$$\begin{pmatrix} \dot{u}_1 \\ \dot{u}_2 \end{pmatrix} = \frac{1}{|\mathbf{A}|} \begin{pmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{pmatrix} \begin{pmatrix} \dot{\phi}(t)/a_1 \\ -\dot{\phi}(t)/a_2 \end{pmatrix}. \tag{20}$$

[13] To prove this, suppose that $a_1 = a_2$ and $u_1 = n \neq u_2 = m$. From (16), $a_1 = a_2$ implies $A_1(n,m) + A_2(n,m) = 0$. Let $\Delta u_1 = (m-n)/2$ and $\Delta u_2 = (n-m)/2$ to get $n' := n + \Delta u_1 = m + \Delta u_2 =: m'$ and therefore $A_1(n',m') + A_2(n',m') = 0$ from (3). Taylor's theorem implies the existence of $(n'',m'') = (n + k\Delta u_1, m + k\Delta u_2)$ for a $k \in (0,1)$ such that

$$\underbrace{A_1(n',m') + A_2(n',m')}_{=0} = \underbrace{A_1(n,m) + A_2(n,m)}_{=0} + [A_{11}(n'',m'') + A_{21}(n'',m'')]\Delta u_1 + [A_{12}(n'',m'') + A_{22}(n'',m'')]\Delta u_2.$$

Since $\Delta u_1 = -\Delta u_2$ and $A_{12}(n'',m'') = A_{21}(n'',m'')$, it follows that $A_{11}(n'',m'') = A_{22}(n'',m'')$, which contradicts assumption (3). Thus, $a_1 = a_2$ implies $u_1 = u_2$.

Because $\dot{\phi}(t) < 0$ and $|\mathbf{A}| < 0$, it is straightforward to show that the assumptions (3) imply that $\dot{u}_1 > 0$, if $A_{12} > -(a_2/a_1)A_{22}$, and $\dot{u}_2 > 0$, if $A_{21} < -(a_1/a_2)A_{11}$. In other words, $\dot{u}_1 > 0$, if A_{12} is not too negative, and $\dot{u}_2 > 0$, if A_{21} is not too positive. Both control variables will be increasing in t, if

$$-\frac{a_1}{a_2}A_{11} > A_{12} > -\frac{a_2}{a_1}A_{22},$$

which, since at least one of the inequalities must be satisfied, moreover implies that at least one of the controls is increasing in time. In the special case with $a_1 = a_2$, we have seen before that $A_{12} = 0$; thus, the inequality is satisfied and $\dot{u}_1$ and $\dot{u}_2$ are positive in this case. In summary, although the first of the three main conclusions of section 3 cannot be definitely answered in the affirmative for the general case, it is approximately valid.

The example employed in section 3 has the special property that $A_{12}(u_1,u_2) = 0$ for all values of (u_1,u_2) and therefore both $\dot{u}_1$ and $\dot{u}_2$ are positive. As an example involving non-vanishing cross partial derivatives, consider the function A given by

$$A(u_1,u_2) = \begin{cases} \frac{1}{2}\sqrt{u_1/u_2} & : \quad u_2 > u_1 \geq 0, \\ 1 - \frac{1}{2}\sqrt{u_2/u_1} & : \quad u_1 > u_2 \geq 0, \\ \frac{1}{2} & : \quad u_2 = u_1 \geq 0. \end{cases}$$

This function fulfills the conditions in (3) but is only C^1 (not C^2) in the positive orthant, however. Substituting the first order derivatives into (16) yields the following Nash equilibrium trajectories:

$$u_1(t) = \frac{1}{4}\sqrt{\tfrac{b_1 b_2}{c_1 c_2 \rho^2}}e^{(1+\rho)(t-T)} \quad \text{and} \quad u_2(t) = \frac{1}{4}\sqrt{\tfrac{c_1 b_2^3}{b_1 c_2^3 \rho^2}}e^{(1+\rho)(t-T)}$$
$$\text{if } a_1 = \tfrac{b_1}{c_1} > \tfrac{b_2}{c_2} = a_2,$$
$$u_1(t) = \frac{1}{4}\sqrt{\tfrac{c_2 b_1^3}{b_2 c_1^3 \rho^2}}e^{(1+\rho)(t-T)} \quad \text{and} \quad u_2(t) = \frac{1}{4}\sqrt{\tfrac{b_1 b_2}{c_1 c_2 \rho^2}}e^{(1+\rho)(t-T)}$$
$$\text{if } a_1 = \tfrac{b_1}{c_1} \leq \tfrac{b_2}{c_2} = a_2.$$

These equilibrium strategies show that the first conclusion of section 3 may be valid even if the cross partial derivatives of A do not vanish.

In order to analyze the validity of the second and third of the three conclusions in section 3, the solution of equation (5) – evaluated at $t = T$ – can be written symbolically as

$$p_1(T) = e^{-T}\left[p_{10} + \int_0^T A(u_1(t),u_2(t))e^t\,dt\right] \tag{21}$$

by the variation of the constant formula. As we have seen before, $u_1(t) \gtreqless u_2(t)$ for all t iff $a_1 \gtreqless a_2$. Since $A(u_1,u_2) \gtreqless 1/2$ iff $u_1(t) \gtreqless u_2(t)$, it follows that

$$\int_0^T A(u_1(t),u_2(t))e^t\,dt \gtreqless \int_0^T \frac{1}{2}e^t\,dt = \frac{1}{2}e^T - \frac{1}{2} \quad \Longleftrightarrow \quad a_1 \gtreqless a_2.$$

We can use this result along with (21) to obtain the second conclusion:

$$p_1(T) \gtreqless \frac{1}{2} + \left(p_{10} - \frac{1}{2}\right) e^{-T} \quad \Longleftrightarrow \quad a_1 \gtreqless a_2,$$

which is exactly what has been deduced from (15). For example, if $p_{10} = 1/2$, then $R1$ is more likely to be preferred at time T if $a_1 > a_2$. Finally, we note that the third conclusion is obviously valid in view of the results obtained so far.

5 Efficiency

In order to evaluate the efficiency of the open-loop Nash equilibrium, it is useful to consider the cooperative solution of joint benefit maximization of both regions. The objective function in this case is

$$\begin{aligned} J = J_1 + J_2 &= \sum_{i=1}^{2} \int_0^\infty [U_i(t) - C_i(u_i(t))]e^{-\rho t}\, dt \\ &= e^{-\rho T}[p_1(T)(b_1 - b_2) + b_2]/\rho - \int_0^T [c_1 u_1(t) + c_2 u_2(t)]e^{-\rho t}\, dt, \end{aligned}$$

and it should be maximized with respect to u_1 and u_2 subject to

$$\dot{p}_1 = A(u_1, u_2) - p_1, \quad p_1(0) = p_{10} \in [0,1].$$

It is well known that the solution of this problem will be Pareto-efficient from the point of view of the two regions, that is, it satisfies the criterion of group-rationality.[14] It should be noted, however, that the outcome of a cooperative game without the possibility of compensating side-payments can be Pareto-efficient in the sense that there is no other outcome for which at least one player gets a larger and no player a smaller payoff, even if it does not solve the problem of joint benefit maximization. If side-payments are possible, however, no solution of the regional competition game is Pareto-efficient unless it is a solution of the joint benefit maximization problem. For J is measured in monetary units, and every solution that does not maximize J is therefore Pareto-dominated by the maximizing solution if compensating side-payments from one region to the other are possible. Therefore, only a solution which maximizes J will be called *efficient* in this section.

In contrast to the non-cooperative Nash equilibrium, for the present case it is important to take the non-negativity constraints explicitly into account right from the beginning because it is likely that one of the u_i should be set equal to zero for all $t \in [0,T]$, regardless of the parameter values. Thus, we have to impose the constraints $u_i \geq 0$, $i = 1,2$.

The current value Hamiltonian is

$$H = -[c_1 u_1 + c_2 u_2] + \lambda[A(u_1, u_2) - p_1],$$

[14]This statement is a minor variation of a standard result on the relation between welfare maximization and Pareto-efficiency, cf. e.g. Varian (1992, p. 333).

and, taking the non-negativity constraints into consideration, the necessary conditions for an optimum include

$$
\begin{aligned}
\frac{\partial H}{\partial u_1} &= -c_1 + \lambda \frac{\partial A}{\partial u_1} \leq 0,\ u_1 \geq 0,\ \frac{\partial H}{\partial u_1} u_1 = 0, \\
\frac{\partial H}{\partial u_2} &= -c_2 + \lambda \frac{\partial A}{\partial u_2} \leq 0,\ u_2 \geq 0,\ \frac{\partial H}{\partial u_2} u_2 = 0, \\
\dot{\lambda} &= \rho\lambda + \lambda, \\
\lambda(T) &= (b_1 - b_2)/\rho.
\end{aligned} \tag{22}
$$

We do not need the explicit solution here because the main conclusions are easily derived from the necessary conditions. From the last two equations of (22) it follows immediately that

$$\lambda(t) \gtreqless 0 \quad \forall t \in [0,T] \quad \iff \quad b_1 \gtreqless b_2.$$

Using this result along with the other relations of (22), the properties (3) of the function A imply that

$$
\begin{aligned}
&(u_1(t) \geq 0 \quad \text{and} \quad u_2(t) = 0 \quad \forall t \in [0,T]) \quad \text{if} \quad b_1 > b_2, \\
&(u_1(t) = 0 \quad \text{and} \quad u_2(t) = 0 \quad \forall t \in [0,T]) \quad \text{if} \quad b_1 = b_2, \\
&(u_1(t) = 0 \quad \text{and} \quad u_2(t) \geq 0 \quad \forall t \in [0,T]) \quad \text{if} \quad b_1 < b_2.
\end{aligned}
$$

Thus, at least one of u_1 and u_2 is zero for all t. If $b_1 = b_2$, it is clearly irrelevant from the point of view of both regions together in which region the facility will be located; therefore, $u_1 = u_2 = 0$ for all t is optimal. If, for example, b_1 is greater than b_2, it may be sensible to try to raise the probability of having the facility located in $R1$. Thus, $u_2 = 0$ and $u_1 \geq 0$ for all t (whether the strict inequality for u_1 will hold depends on the parameter values).

These results indicate that the open-loop Nash equilibrium is highly inefficient from the point of view of both regions together and therefore for the inhabitants of these regions. For example, if $b_1 = b_2$, the joint benefit maximization requires to spend nothing on exerting influence, while these expenditures are likely to be positive in both regions in the Nash equilibrium (cf. the discussion following equation (13)). This result resembles that of Asada (1997) for the case of the transportation competition, who claims that the competition between two firms is not necessarily inefficient compared to the case of cooperation from the social point of view because the expenditures of his firms will improve the quality of the transportation means. In the present case, however, this line of argument is not valid: Exerting influence of a region in order to raise its own attraction with respect to the LDM involves costs by definition; so far as these actions would have a value by themselves, rational regional governments would carry them out without regard to the possible location of a new facility. The costs of influence should therefore be interpreted as net costs that have no direct compensation in terms of the utility for the region's inhabitants. Thus, these costs have to be subtracted from the benefit provided by the new facility.

On the other hand, the LDM would ignore the expenditures of the regions if they would not be beneficial to him. From the point of view of the two regions and the LDM together, the LDM's extra benefit has to be taken into account. Apart from the distributional problem involved, however, it is most likely that the gain of the LDM does not outweigh the loss of the regions.

In summary, from a social point of view, the regions should not compete for the LDM but should solve their problem of joint benefit maximization – the solution of which may be that it is optimal to do nothing at all –, wait for the LDM's decision and come to an agreement on the payments that the preferred region passes to the loosing region. (By the way, this is an argument in favor of the German *Länderfinanzausgleich*.)

6 Concluding Remarks

We have dealt with regional competition for the location of new facilities in the framework of a differential game, the simplicity of which makes possible its quantitative and/or qualitative solution. Moreover, state-separability implies that the unique open-loop Nash equilibrium of the model is also a degenerate subgame perfect feedback equilibrium. Despite this simplicity, the model seems to be reasonably well suited for analyzing the problem at hand, thereby providing interesting insights into this process of regional competition.

The concept of a Nash equilibrium is sensible if both regions play symmetrical roles with symmetrical information structures. A possible extension of the model analyzed here is the consideration of the von Stackelberg equilibrium, with one region being the leader and the other being the follower. This setting, where the leader informs the follower about his own strategy in advance, may be a realistic description of some actual competition processes. Note that in order to obtain a reasonable von Stackelberg equilibrium it is not possible to use the specific example of section 3, because the function A employed there uncouples the decision processes of both players. The functional form used as an example in section 4 does not exhibit this property, however.

Another possible extension is the explicit analysis of the LDM as a third player who tries to influence the actions of the regions by giving feedbacks about special requirements important for his decision process. The analysis of these interactions belongs to the primary purposes of the model that Jutila (2001) has in mind. The formulation of an adequate differential game, however, would be much more complicated than the present model, because discrete time elements would have to be integrated into the continuous time model. For the LDM would likely give his feedbacks only from time to time. Moreover, three player non-symmetrical differential games are generally difficult to analyze. It should be noted, however, that although we have not introduced the LDM as a third player in the game, he nevertheless has some influence on the ongoing competition process. For the LDM's preferences must be understood to have consequences with respect to the functional form of A as well as to the levels of the cost parameters c_i and the initial probabilities p_{i0}.

The introduction of more than two regions competing for the location of a new facility would probably be more easy to accomplish, at least as long as all regions play symmetrical roles and differ only with respect to the parameters b_i and c_i. However, one should not expect too much of this kind of extension. The basic insights are likely to be the same as those from the two regions model, although some qualifications with respect to the numerical values of the optimum strategies will be involved. This depends on the functions replacing A in a multi player setting. Beyond this qualification, the results of the two player model should be reasonably robust with respect to the introduction of many players because the basic structure of the model is not affected by this generalization.

Finally, it would be interesting to analyze how an efficient outcome of the kind considered in section 5 could be reached by cooperative modes of play with possible side-payments. With regard to the competing regions, this is the most important question from the practical point of view. It should be noted that in general the cooperative solution of joint benefit maximization as analyzed in section 5 cannot be accomplished without side-payments. For if $b_1 > b_2$, e.g., this solution usually requires to raise $p_1(T)$ with $u_1 \geq 0$ and $u_2 = 0$. Thus, in the absence of compensating side-payments, region 2 would probably lose (in terms of expected benefit) in comparison to the non-cooperative open-loop Nash strategy. Since the benefits of the location of a facility do not arise before the end of the game (considering its extensive form), the determination of appropriate modes of side-payments is all but an easy task. How could one region trust in the promise of the other region to divide the profit of the localization project amongst each other when there is no possibility of punishment during the time the game is played? Without the possibility of compensating side-payments, two approaches are discussed in the literature. The direct extension of the Nash bargaining solution with variable threat points to differential games involves the maximization of a weighted average of the regions' objective functions, with the weights being determined as part of the solution. In the sense defined in section 5, the solution would be Pareto-efficient, but in general not efficient. The problem with the approach is that the threat strategies are usually not credible. Alternatively, one might try to solve the game using subgame perfect trigger strategies with the non-cooperative open-loop equilibrium strategies as credible threats in order to stabilize a Pareto-efficient outcome. Since we are considering a finite-time game, however, such equilibria do not exist in case of a fixed positive time delay between defection and punishment, which necessitates the use of so-called *δ-strategies*.[15] We must leave a thorough analysis of such almost cooperative solutions for future research.

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[15] Cf. Tolwinski et al. (1986).

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