Economic Crises, Housing Price Bubbles, and Saddle-Point Economics

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Abstract. Starting from the recent financial and economic crisis the standard interpretation of rational expectations and optimal control dynamic models is called into question referring to its lack of robustness. Using the example of the housing market it is shown that an alternative interpretation closer to the mathematical basics would imply economies much more crisis-prone than they actually are. Market frictions reduce chances for bubbles and should be taken into consideration in positive as well as in normative economics. Dynamic models in a neoclassical spirit can explain reality only with frictions, and increasing frictions may sometimes be a reasonable stabilizing instrument of economic policy.

JEL-Classification: B40, C61, D90

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1 Saddle Points in Dynamic Economics

The economics profession is criticized for not having been aware of the emergence of the financial and economic crisis 2009–2010. According to Colander et al. (2009) the lack of understanding is due to the profession's insistence on using models that disregard the key elements driving outcomes in real-world markets. Among other issues they especially stress that robustness of economic models should be a key concern.

According to Kirchgässner (2009) the criticism on economics falls into three major categories. First, economists failed to reasonably assess the risks accompanying the issue of collateralized debt obligations, the possible consequences of a bursting housing bubble, and so on. Second, the ongoing debate about the orientation of economic theory towards more ore less mathematics and its concept of rational behavior has been reinforced. Third, the principle of impartiality in the social sciences is called into question. Kirchgässner (2009) concedes that the first point is a just criticism but challenges the second and third one. The present article mainly contributes to the second point and argues that the concept of rationality has to be revised in dynamic economic models. By relating to the robustness issue addressed by Colander et al. (2009) it will also be shown that the failure to be aware of the risks of an upcoming crisis can partly be explained by an interpretation of dynamic models in economics that is not robust.

Stiglitz (2011) indicates several possible reasons for the failure of the *Conventional Wisdom*, as he calls the implications of the dynamic stochastic general equilibrium model of macroeconomics described for example in Gali (2008). These issues are mostly related to neglecting insights from modern microeconomics in modern macroeconomics, such as the lack of efficiency in case of asymmetric information and other sources of market failure. The focus here is on an issue that is also briefly discussed by Stiglitz (2011, p. 617): "There has been a shift in ... what is meant by a stable system", whereby he addresses the so-called *saddle-point stability* in rational expectations and optimal control dynamic models. The present paper argues that this redefinition of stability is partly responsible for indeed pushing the concept of rationality too far. There has been some debate about this issue from the late 1960's to the early 1980's, but afterwards the redefinition of stability seems to have become more or less accepted in mainstream economics. The first part of this article therefore reviews some of the old arguments.

As the differential equations in rational expectations and optimal control models usually are (perturbed) Hamiltonian systems, the theorems of Poincaré and Kurz (cf. Kurz, 1968) imply that their long-run equilibria regularly are saddle points. Mathematically speaking, saddle points are unstable, not stable equilibria. The standard interpretation in economics is, however, that these stationary equilibria will be asymptotically reached along the stable arms of the saddle paths, which at least requires extreme information on the side of economic agents who have to choose initial conditions of some jump variables with infinite accuracy. That interpretation precludes economic crises since during this movement the market is always in short-run equilibrium and it always approaches the efficient long-run equilibrium.

In order to avoid confusion about the various concepts of *equilibrium* used in this paper, they are defined as follows: A long-run or stationary equilibrium is a stationary point of the model's differential equations. A market equilibrium is a situation where demand and supply are equalized at every point in time and, in case of a perfect foresight competitive equilibrium, agents have correct expectations about the future. A stationary equilibrium is usually also a market equilibrium, but not the other way around. In fact, the saddle-path in rational expectations models itself constitutes the market equilibrium. Such theories are, strictly speaking, therefore only concerned with the equilibrium itself and not with the adjustment towards it in case of a disequilibrium. In this sense it is not correct to call the market equilibrium unstable, because the theories have nothing to tell about its stability or instability. But things are even worse: No convincing adjustment process to the market equilibrium has ever been proposed,[1] and the market equilibrium itself moves along a knife-edge path unstable in relation to the long-run equilibrium.

Two of the best known models belonging to this class of saddle-path rational expectations models can be found in Dornbusch (1976) and in the seminal paper by Sargent and Wallace (1973). The latter introduced the idea to replace the usual *backward* solution of an unstable differential equation by a stable *forward* solution. Math-

[1] To the best of my knowledge the only attempt to formulate such an adjustment process is due to Heller (1975), who introduced future markets and proved stability under standard assumptions such as gross substitutability. However, this procedure amounts to nothing more than eliminating the original equilibrium dynamics of the model since all trading is done in advance on the future markets, thereby getting room for disequilibrium dynamics.

ematically this amounts to letting one of the variables (the only endogenous one in Sargent and Wallace, 1973) jump exactly on the stable arm of the saddle point to meet an end-point condition (instead of an initial condition met by the backward solution). Economically the underlying assumption is that agents have rational expectations (or perfect foresight in a deterministic context), expect that every dynamic process reaches a long-run equilibrium, and hit the necessary value of the jump variable with infinite accuracy.

It did not take long before critics questioned this process of arbitrary selection of the saddle-path solution. A possible remedy came from the related theory of optimal-control models. If it was possible to interpret the rational expectations models as having an underlying framework of intertemporal optimization, the selection of the stable arm could be part of the optimum conditions and therefore of the market equilibrium. An important example in this respect is Brock (1975), which in a continuous time version is also discussed in Gray (1984). This author shows, however, that Brock's work does not generally justify the arbitrary selection of the stable arm. Instead, not only the stable arm of the stationary equilibrium of the differential equations can be a market equilibrium solution, but in the general case there is also an infinity of implosive real balance paths that all constitute a market equilibrium under rational expectations (Gray, 1984, p. 101). As Blanchard and Fischer (1989, p. 260) put it: "These divergent solutions can *sometimes* be ruled out by partial or general equilibrium arguments, although the arguments often rely on a degree of rationality and foresight that is unlikely to be present in practice" (emphasis added by author). But (at least) in cases where the divergent paths cannot be ruled out, the crises-free saddle-path interpretation is not robust. Instead, the robust interpretation would be that deviations from the saddle path are the rule, not the exception. These deviating paths, however, would eventually lead to economic crises.

The problem of saddle-point instability was first addressed by Hahn (1966) in a heterogeneous capital goods model. This *Hahn problem* potentially arises in any dynamic model in which agents may hold their wealth in alternative assets (e.g., money and capital) and have myopic perfect foresight. To be clear about the various expectations hypotheses, their definitions are recalled. Let $P(t)$ and $\dot{P}(t)$ be a price at time t and its instantaneous rate of change at time t, respectively. Then, denoting expectations by $E(.)$, *myopic perfect foresight* means that $E(P(t)) = P(t)$ and $E(\dot{P}(t)) = \dot{P}(t)$ for all t. If in addition the complete time path of $P(t)$ is known in advance including the knowledge that it does not *explode* or *implode* (that is, that it stays on a stable saddle path), the agents have *perfect foresight*. The difference between the two concepts is that under perfect foresight agents know the transversality conditions and accordingly initially always start on the stable saddle path by choosing the correct value of the jump variable, although the theory is silent about how real world agents should be able to accomplish this. Both concepts are used as the equivalent to rational expectations in a deterministic context.

A general proof of instability for a wide class of models has been provided by Kurz (1968), who showed that even *utility* as a second asset in the standard neoclassical growth model generates instability. In short, the theory of economic growth suffers from an instability problem similar to that of monetary macroeconomics. A possible remedy similar to the one just described again comes from dynamic optimization. If agents are completely informed about the future development of economic variables, a decentralized perfect foresight competitive equilibrium can be shown to be equivalent to a problem of optimal economic growth under suitable conditions (Becker, 1981). In such a case exploding paths can be excluded by transversality conditions (e.g. those of Benveniste and Scheinkman, 1982) that are part of the general optimum conditions, while imploding paths can be ruled out as a market equilibrium if they violate some non-negativity condition. Burmeister (1980, S. 234), however, puts it as follows: "If we are willing to ignore reality, a ... theoretically satisfactory solution to the Hahn saddlepoint instability problem has been provided ... ".

It should be added that accepting dynamic optimization as a satisfactory solution to the instability problem actually requires us also to ignore the importance of robustness in model building (on the general concept of robustness cf. Weisberg, 2006, e.g.). Even if choosing the saddle path can be shown to be necessary in terms of dynamic optimization and therefore also for a path to constitute a market equilibrium with optimizing agents, the slightest error in determining the initial conditions required to hit the saddle path will take the actual solution more and more away from it. Mathematicians have had a good reason to call saddle points *unstable*.[2] If it really was reasonable to assume that real-world agents were able to hit the saddle path with perfect accuracy, why must excellent economists allocate their time to the development of methods for calculating such paths (e.g. Brunner and Strulik, 2002; Trimborn et al., 2008)?

Returning to the introductory statement referring to Colander et al. (2009), the major problem in model building seems not to be that key elements of real-world markets are disregarded. For it is the very nature of models that they are extremely simplified images of the real world. What matters most, however, is that the implications of these simplified images should not depend too sensitively on special assumptions about parameters, agents' abilities to calculate and forecast and so on, that is, the models should be robust in this respect. It appears that this requirement of robustness, stressed also by Colander et al. (2009), is ignored too often. E.g., positive interpretations of dynamic optimization models with saddle-path dynamics have become the unquestioned standard in growth theory at least since the seminal papers on endogenous growth by Romer (1986, 1990) and Lucas (1988) have been published, cf. e.g. Barro and Sala-i-Martin (2003) and most of the recent theoretical publications on economic growth.[3] Part of the reason is that the concept of rational behavior mentioned by Kirchgässner (2009) as one of the issues emphasized by critics of economics has been pushed too far. While in static models optima may be approached by rules of thumb if agents are not able to calculate them (cf. e.g. Baumol and Quandt, 1964), such rules of thumb are scarcely known for dynamic optimization problems. In fact, as the market equlibrium itself in rational expectations models constitutes a dynamic path, any kind of adjustment process towards such a path would have to be *metadynamic* in nature. But in reality, we do not have time to approach time.

If the saddle-point models were an adequate description of reality and if one

[2]Numerical examples as well as a discussion of the related problem of open loop versus feedback solutions can be found in Christiaans (2001).

[3]By the way, the lack of robustness in modern growth theory does not only occur with respect to saddle-path instability but also with respect to the knife-edge conditions for endogenous growth (cf. Christiaans, 2004).

agreed that agents will not be able to hit the stable arms with infinite accuracy, they would predict repeated economic crises, although they would not be useful in determining the exact point in time where a particular crisis begins. These issues will be explained in section 2.1 referring to a model of the housing market introduced by Poterba (1984) and Mankiw and Weil (1989). Section 2.2 uses methods similar to those in Burmeister and Turnovsky (1978) to overcome instability by introducing frictions into the market. These sections mainly serve as an illustration of the arguments set forth so far. These arguments are taken up again in the final section 3, which draws the major conclusions from the analysis and also discusses some possible objections to the views expressed in this paper (robust control and adaptive learning).

2 An Illustration: House Price Dynamics

2.1 Rational Expectations and Instantaneous Market Clearance

In view of the origins of the recent economic and financial crisis in the U.S. housing market, the rational expectations model of the housing market analyzed by Poterba (1984) and Mankiw and Weil (1989) suggests itself as an illustration of the preceding arguments. The purpose of this section is not to judge the plausibility of the model as such. It just serves as an example of the common interpretation of saddle-point dynamics, which in the author's opinion should be changed.

The reduced form of the model contains two dynamic equations. The symbols used are the same as those in Mankiw and Weil (1989). The first equation captures the dynamics of the real price P of a standardized unit of housing and is the result of a no-arbitrage condition implying that agents are indifferent between renting or buying a house:

$$\dot{P} = rP - R(h), \tag{1}$$

A dot over a variable designates its time derivative, h is housing per adult, rP is the operating cost of owning a house of price P (interest, taxes, maintenance and depreciation), and $R(h)$ is the market-clearing rental rate. This rental rate equilibrates demand for housing per adult, $h^d = f(R)$ with $f'(R) < 0$, and the stock of housing per adult, h. Thus, in equilibrium, $R(h)$ is just the inverse of $f(R)$ and the rental rate depends negatively on housing per adult, $R'(h) < 0$.

The second equation captures the dynamics of housing per adult, h, and emanates from gross investment in housing:[4]

$$\dot{h} = \psi(P) - (n+\delta)h, \tag{2}$$

where n is the population growth rate, δ the rate of depreciation, and $\psi(P)$ is part of the gross investment function, assumed to depend positively on the price, $\psi'(P) > 0$.

The two zero isoclines obtained by setting $\dot{P} = 0$ and $\dot{h} = 0$ are illustrated for a linear case in figure 1, which illustrates the standard RE-interpretation of the housing model. As the plus- and minus-signs indicate, the long-run equilibrium where

[4] For the derivation of this *intensity*-equation from the original equation in absolute, not per adult variables, cf. Mankiw and Weil (1989).

both variables are constant is a saddle point. If, due to whatever reason, housing per adult h_0 is below its long-run equilibrium value at the intersection of the isoclines $\dot{P} = 0$ and $\dot{h} = 0$, the stationary equilibrium would not be reached unless the housing price jumps on the stable arm of the saddle point by exactly taking on the value P_0. The market then moves along the stable arm and asymptotically approaches the long-run equilibrium. The price of a housing unit declines as housing per adult increases. The rational expectations literature *assumes* that such a jump on the stable arm occurs with infinite accuracy (cf. also Burmeister and Wall, 1982, p. 255). Agents *choose* P_0 because they expect that the price converges to its long-run equilibrium (perfect foresight). The theory is silent, however, about the mechanism that could accomplish such a difficult task, especially in a setting of perfect competition where the price is assumed to be exogenous to individual agents.

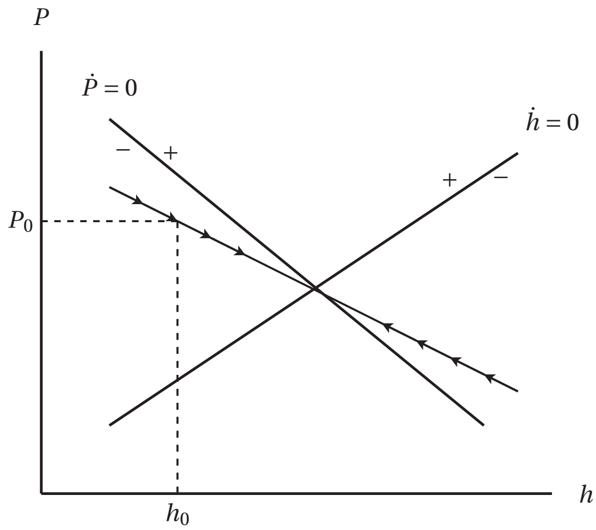

Figure 1. Saddle Path-Dynamics under Perfect Foresight

Figure 2 illustrates the same market under the assumption that the jump variable P misses its saddle-path value by an arbitrary small amount. Under myopic perfect foresight, agents do not know the correct end-point conditions. As the equilibrium is a saddle point, the actual path diverges more and more from the stable saddle path and eventually enters the region with increasing housing prices constituting a speculative bubble. Along this path the price of a housing unit increases as housing per adult increases.

The basic arguments in the rational expectations literature to rule out such explosive paths have been reviewed in section 1. In short, it is claimed that such a path cannot be a market equilibrium with optimizing agents having rational expectations. There are important arguments, however, why this assertion may not hold. First, Blanchard (1979) has shown (admittedly using a model rather different from the present one) that bursting price bubbles may be compatible with full rationality if investors in each period expect a crash of the bubble with some probability $0 < \pi < 1$. The bubble then will end with probability one in the future but it is perfectly rational to follow the bubble path. That is, when agents recognize they are on an exploding bubble, it could still be rational to stay in the market to try and await a

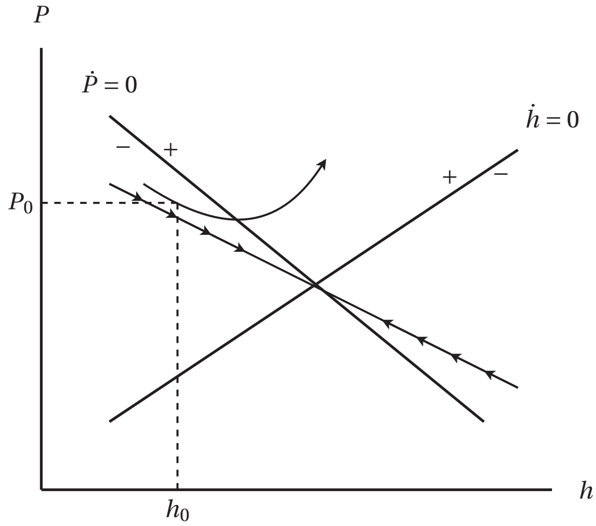

Figure 2. Speculative Bubble under Myopic Perfect Foresight

favorable occasion to leave the market.[5] Second, Gray (1984) has shown that a general exclusion of non-convergent paths is not possible even if there is an underlying optimizing framework because this exclusion depends on the special structure of the model. Third, one may doubt whether the degree of rationality that real-world agents possess really suffices to calculate the correct initial conditions to hit the saddle path. It is important in this regard to observe again that at the moment when agents recognize they are on a bubble, for whatever reason, it could always be perfectly rational to follow this path for some time in order to profit from the price movements along the bubble.

So now, which of the outcomes is more likely? The saddle point is definitely unstable; from a purely mathematical point of view the probability of hitting the stable arm has measure zero if there is the slightest inaccuracy in calculations. If equations (1) and (2) were a reasonable description of market dynamics, it had to be expected that a bubble path (or an imploding path) started repeatedly. But if agents recognize they are on a bubble, it may be perfectly rational to stay on it for some time rather than trying to hit the saddle path again. This appears to be a reasonable description of the origin of the financial crisis in 2008.

2.2 Frictions in the Housing Market

The purpose of this section is to apply methods similar to those of Burmeister and Turnovsky (1978) to the housing market just analyzed. In a word, their idea is to eliminate the saddle-point instability by introducing two kinds of *frictions* into the market. According to their analysis instability is due to the assumptions of myopic perfect foresight and instantaneous market clearance. Changing both assumptions amounts to introducing sufficiently stabilizing frictions into the market. It will be shown how a method similar to theirs can be used to get a stable long-run equilib-

[5] An early example of such a rational bubble has been provided by Samuelson (1957) referring to the Dutch tulip mania 1636–37.

rium of the housing market. As Burmeister and Turnovsky (1978) were concerned with the capital market, some modifications are necessary to adapt their analysis to the housing market.

First note that the no-arbitrage condition (1) has already been formulated as an equilibrium condition where expected and actual price changes are equal and where in addition $R(h)$ is the market-clearing rental rate. More generally, both conditions will not always be met. Starting more from the beginning, the equation must read

$$E(\dot{P}) = rP - R(h^d),$$

where $E(\dot{P})$ is the expected price change and $R(h^d)$ is the inverse of the demand for housing function $h^d = f(R)$. Burmeister and Turnovsky (1978, p. 292) derive the following hypotheses about expectations as a limiting continuous time version of a general discrete time adaptive expectations equation, where γ is constant:

$$E(\dot{P}) = \gamma \dot{P}$$

For obvious reasons they assert that $\gamma < 0$ corresponds to irrational, $\gamma = 0$ to static, $0 < \gamma < 1$ to weakly rational, and $\gamma = 1$ to rational expectations (myopic perfect foresight). It will now be assumed that $0 < \gamma < 1$. After rearranging, the new no-arbitrage condition under weakly rational expectations is therefore

$$R(h^d) = rP - \gamma \dot{P}, \quad 0 < \gamma < 1.$$

Since $R(h^d) = f^{-1}(h^d)$, this equation can be solved for h^d to get

$$h^d = f(rP - \gamma \dot{P}). \tag{3}$$

The model in section 2.1 assumes permanent market clearing. For $R = R(h)$ is the market-clearing rental rate given housing per adult h emanating from the equilibrium condition $f(R) = h$. More generally, the market will not always be in equilibrium and it is necessary to formulate an adjustment process for the case that h^d according to (3) does not equal the supply of housing, h. Following Burmeister and Turnovsky (1978) it will analogously be assumed that[6]

$$\dot{P} = \alpha[h^d - h], \quad \alpha > 0.$$

The larger α, the higher is the adjustment speed, and the smaller α, the larger are the frictions in the market. Substituting (3) for h^d yields

$$\dot{P} = \alpha[f(rP - \gamma \dot{P}) - h]. \tag{4}$$

As it has been assumed that there is a unique equilibrium where $f(R) = h$, the implicit function theorem yields the existence of a function

$$\dot{P} = g(P, h)$$

[6]Actually, Burmeister and Turnovsky (1978) would assume that $\dot{P}/P = \alpha[f(R) - h]$. The hypothesis adopted here turns out to be easier handled than their original hypothesis applied to the present model. An additional difference arises from the fact that in their asset-market model demand depends on r instead of R, which would not be reasonable in the housing-market model considered here.

in the vicinity of the equilibrium where $\dot{P} = 0$, provided that the derivative of $\dot{P} - \alpha[f(rP - \gamma\dot{P}) - h] = 0$ with respect to $\dot{P}$ does not vanish:

$$1 + \gamma\alpha f' \neq 0 \quad \text{or} \quad f' \neq -\frac{1}{\gamma\alpha}.$$

Let this condition be met. The implicit function theorem then implies that the partial derivatives of $g(P, h)$ are

$$\frac{\partial\dot{P}}{\partial P} = \frac{\alpha r f'}{1 + \gamma\alpha f'}$$
$$\frac{\partial\dot{P}}{\partial h} = \frac{-\alpha}{1 + \gamma\alpha f'}$$

Equation (2) remains unaltered. Thus, the new system of differential equations replacing (1) and (2) is

$$\dot{P} = g(P, h) \tag{5}$$
$$\dot{h} = \psi(P) - (n + \delta)h \tag{6}$$

with Jacobian

$$J = \begin{pmatrix} \dfrac{\alpha r f'}{1 + \gamma\alpha f'} & \dfrac{-\alpha}{1 + \gamma\alpha f'} \\ \psi' & -(n + \delta) \end{pmatrix}$$

The Routh-Hurwitz conditions for local asymptotic stability of the long-run equilibrium read:

$$\frac{\alpha r f'}{1 + \gamma\alpha f'} - (n + \delta) < 0$$
$$-\frac{\alpha r f'}{1 + \gamma\alpha f'}(n + \delta) + \frac{\alpha}{1 + \gamma\alpha f'}\psi' > 0$$

Suppose that $1 + \gamma\alpha f' < 0$. As $f' < 0$ and $\psi' > 0$, the second Routh-Hurwitz condition would be violated and the long-run equilibrium would still be a saddle point. If $1 + \gamma\alpha f' > 0$, however, it is straightforward that both Routh-Hurwitz conditions are met. In other words: If γ and/or α is sufficiently small (the frictions are sufficiently large), the housing market becomes at least locally stable.

Now let $1 + \gamma\alpha f' > 0$. Depending on the size of the discriminant of J, the stationary equilibrium of system (5), (6) can either be a stable node or a stable spiral. As the trace of J and its determinant both do not vanish if $1 + \gamma\alpha f' > 0$, both eigenvalues have nonzero real part and the Hartman-Grobman theorem implies that even in the nonlinear case the possible equilibria are stable nodes and spirals. Figure 3 shows the case of a stable spiral calculated for a linear example.[7]

[7] The functions and parameters used for this example are: $f(R) = 3.2 - R$, $\alpha = 1$, $\gamma = 0.5$, $r = 0.1$, $\psi(P) = P - 1.7$, $n = \delta = 0.05$. Substituting into (5) [or (4)] and (6) yields $\dot{P} = -0.2P - 2h + 6.4$ and $\dot{h} = P - 0.1h - 1.7$. This system has its unique equilibrium at $(P, h) = (2, 3)$.

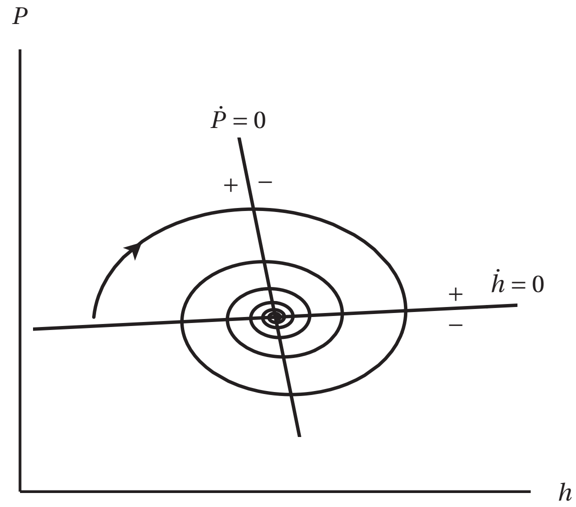

Figure 3. Stable Spiral due to Frictions

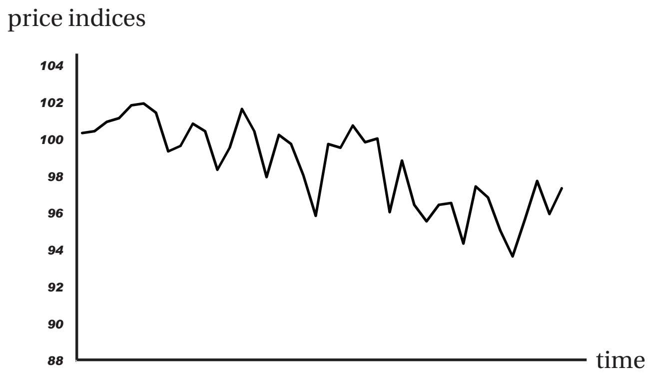

Figure 4. House Price Indices Germany 1st Quarter 2000 to 4th Quarter 2009, Used Houses, Data Source: Statistisches Bundesamt, Wiesbaden 2011

Although this model appears to be too simplified for an unamended application to real world data, the dynamic evolution just described has some appeal when compared with actual house prices time series. Figure 4 is just one example showing the development of the price indices in Germany for used houses from 2000 to 2009. Starting from an equilibrium of equations (5) and (6), a decline in the population growth rate would lead to a new equilibrium with a lower price and a higher value of housing per adult (the slope of $\dot{h} = 0$ would decrease). The spiraling approach to the new equilibrium would indeed generate a time series of housing prices comparable to that in figure 4. It should also be observed that such a cyclical price movement cannot be explained by the original rational expectations model in the absence of stochastic shocks.

Before closing this section a final comment on uniqueness of solution paths is in order. One of the perceived advantages of saddle-point equilibria in the rational expectations literature is the uniqueness of the price level that follows from choosing the saddle path. As the model with frictions may be stable for all initial values, uniqueness is lost, of course. This indeterminacy of the price (or price level in aggregated models) in models with stable equilibria may be one of the reasons why saddle-path models are so popular (cf. also Gray, 1984, p. 116). But the uniqueness-advantage should not be a sufficient reason to stick to models that are not robust, all the more as the theory has nothing to tell about how competitive agents should manage to jump on the saddle path.

3 Conclusion

Theories asserting that economies converge along a saddle path towards the stationary equilibrium are not robust. This holds true for the case of aggregate models in macroeconomics as well as for single market models such as the housing model considered in section 2. If the differential equations involved were a reasonable image of reality, a robust implication would be that crises due to exploding price paths would be the rule, not the exception. It should be stressed that this assessment needs no empirical validation but follows from methodological considerations. From a purely mathematical point of view saddles are unstable, and considering the requirement of robustness, unstable equilibria cannot seriously be used as a regular prediction of real-life phenomena.

Advocates of the mainstream rational expectations model have made some effort themselves in relaxing the extreme requirements inherent in the assumption of perfect foresight. As the question arises whether these extensions of the standard theory make the present critical assessment obsolete, some comments are necessary. One of those amendments is the theory of robust control (cf. Hansen and Sargent, 2007). At first sight, the notion "robust control" seems to imply that the associated solutions do also work in case that the initial values miss the saddle path. But that is not the case. Robust control is only robust in the limited sense that it is taken into account that true parameter values of the economy may be unavailable to the planner and so he decides to minimize the maximum expected loss in calculating the optimum for the worst case scenario, e.g. The solution thus calculated will suffer from the same lack of robustness as the solutions described in the present paper.

Another possible remedy comes from the theory of adaptive learning (cf. Evans and Honkapohja, 2001). It is true that the long-run equilibrium in case of adaptive learning may be stable in the original mathematical sense. However, adaptive learning dynamics are not too far away from dynamics with market frictions described in section 2.2, e.g. Turning a saddle path into a stable node or spiral implies that the model and its implications for market efficiency (which is lost) are substantially altered. In other words, adaptive learning models are another possible solution to the problems described here but they are far away from replacing perfect foresight models as the mainstream theory.

Although the author has always been convinced that saddle paths do not provide robust implications, he does not claim he had been able to predict the economic and financial crisis 2009–2010. The interpretation was always the other way around: As we are not living in a world where economic bubbles rule, the author concluded that these models were no adequate description of reality. Rather, there would have to be some kind of friction that tends to make unstable paths stable (as in section 2.2), or people would have to base their expectations and decisions on completely different grounds.

But the analysis in section 2.2 has also shown that instability is ruled out only provided that frictions are sufficiently large. If frictions are decreased, perhaps due to electronic exchange at international stock markets, the chances for bubbles will increase, making the world economy more crisis-prone. The economic and financial crisis 2009–2010 provides evidence for this assessment.

The analysis of this paper implies, first, that there are frictions in the markets that economists should take into consideration when building economic models (without frictions economic crises would happen more often), and second, that frictions are not only detrimental to an economy but may help to prevent speculative bubbles. Of course, that idea is not new but goes back to Tobin (1978, p. 154) when he proposed "to throw some sand in the wheels of our excessively efficient international money markets", although the reasoning in the present paper is different from his.

References

Barro, R. J. and Sala-i-Martin, X. (2003): *Economic Growth*, 2nd edition, MIT Press, Cambridge.

Baumol, W. J. and Quandt, R. E. (1964): Rules of Thumb and Optimally Imperfect Decisions, *American Economic Review*, 54, pp. 23–46.

Becker, R. A. (1981): The Duality of a Dynamic Model of Equilibrium and an Optimal Growth Model: The Heterogeneous Capital Goods Case, *Quarterly Journal of Economics*, 95, pp. 271–300.

Benveniste, L. M. and Scheinkman, J. A. (1982): Duality Theory for Dynamic Optimization Models of Economics: The Continuous Time Case, *Journal of Economic Theory*, 27, pp. 1–19.

Blanchard, O. and Fischer, S. (1989): *Lectures on Macroeconomics*, MIT Press, Cambridge.

Blanchard, O. J. (1979): Speculative Bubbles, Crashes and Rational Expectations, *Economics Letters*, 3, pp. 387–389.

Brock, W. A. (1975): A Simple Perfect Foresight Monetary Model, *Journal of Monetary Economics*, 1, pp. 133–150.

Brunner, M. and Strulik, H. (2002): Solution of Perfect Foresight Saddlepoint Problems: A Simple Method and Applications, *Journal of Economic Dynamics and Control*, 26, pp. 737–753.

Burmeister, E. (1980): *Capital Theory and Dynamics*, Cambridge University Press, Cambridge.

Burmeister, E. and Turnovsky, S. J. (1978): Price Expectations, Disequilibrium Adjustments, and Macroeconomic Price Stability, *Journal of Economic Theory*, 17, pp. 287–311.

Burmeister, E. and Wall, K. D. (1982): Kalman Filtering Estimation of Unobserved Rational Expectations with an Application to the German Hyperinflation, *Journal of Econometrics*, 20, pp. 255–284.

Christiaans, T. (2001): Economic Growth, the Mathematical Pendulum, and a Golden Rule of Thumb, *Discussion Paper No. 94-01*, University of Siegen.

Christiaans, T. (2004): Types of Balanced Growth, *Economics Letters*, 82, pp. 253–258.

Colander, D., Föllmer, H., Haas, A., Goldberg, M., Juselius, K., Kirman, A., Lux, T., and Sloth, B. (2009): The Financial Crisis and the Systemic Failure of Academic Economics, *Kiel Working Paper 1489*, Kiel.

Dornbusch, R. (1976): Expectations and Exchange Rate Dynamics, *Journal of Political Economy*, 84, pp. 1161–1176.

Evans, G. W. and Honkapohja, S. (2001): *Learning and Expectations in Macroeconomics*, Princeton University Press, Princeton.

Gali, J. (2008): *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press, Princeton.

Gray, J. A. (1984): Dynamic Instability in Rational Expectations Models: An Attempt to Clarify, *International Economic Review*, 25, pp. 93–122.

Hahn, F. H. (1966): Equilibrium Dynamics with Heterogeneous Capital Goods, *Quarterly Journal of Economics*, 80, pp. 633–646.

Hansen, L. P. and Sargent, T. J. (2007): *Robustness*, Princeton University Press, Princeton.

Heller, W. P. (1975): Tâtonnement Stability of Infinite Horizon Models with Saddle-Point Instability, *Econometrica*, 43, pp. 65–80.

Kirchgässner, G. (2009): Die Krise der Wirtschaft: Auch eine Krise der Wirtschaftswissenschaften?, *Perspektiven der Wirtschaftspolitik*, 10, pp. 436–468.

Kurz, M. (1968): The General Instability of a Class of Competitive Growth Processes, *Review of Economic Studies*, 35, pp. 155–174.

Lucas, Jr., R. E. (1988): On the Mechanics of Economic Development, *Journal of Monetary Economics*, 22, pp. 3–42.

Mankiw, N. G. and Weil, D. N. (1989): The Baby Boom, the Baby Bust, and the Housing Market, *Regional Science and Urban Economics*, 19, pp. 235–258.

Poterba, J. M. (1984): Tax Subsidies to Owner-Occupied Housing: An Asset-Market Approach, *Quarterly Journal of Economics*, 99, pp. 729–752.

Romer, P. M. (1986): Increasing Returns and Long Run Growth, *Journal of Political Economy*, 94, pp. 1002–1037.

— (1990): Endogenous Technological Change, *Journal of Political Economy*, 98, pp. S71–S102.

Samuelson, P. A. (1957): Intertemporal Price Equilibrium: A Prologue to the Theory of Speculation, *Weltwirtschaftliches Archiv*, 79, pp. 181–219.

Sargent, T. J. and Wallace, N. (1973): The Stability of Models of Money and Growth with Perfect Foresight, *Econometrica*, 41, pp. 1043–1048.

Stiglitz, J. E. (2011): Rethinking Macroeconomics: What Failed, and How to Repair It, *Journal of the European Economic Association*, 9, pp. 591–645.

Tobin, J. (1978): A Proposal for International Monetary Reform, *Eastern Economic Journal*, 4, pp. 153–159.

Trimborn, T., Koch, K.-J., and Steger, T. M. (2008): Multidimensional Transitional Dynamics: A Simple Numerical Procedure, *Macroeconomic Dynamics*, 12, pp. 301–319.

Weisberg, M. (2006): Robustness Analysis, *Philosophy of Science*, 73, pp. 730–742.