On the Dynamics of Competition between Commercial and Free Software

Thomas Christiaans
FOM University of Applied Sciences, 57078 Siegen, Germany
thomas.christiaans@fom.de

This is the pre-peer reviewed version of the following article: *On the Dynamics of Competition between Commercial and Free Software*. The final, definitive version of this paper has been published in *Studies in Microeconomics* 1 (1), 2013, pp. 37–58, by SAGE publications India Pvt. Ltd.

Abstract. This paper introduces dynamic network externalities into a Hotelling-like model of competition between commercial and free software. The assumption of linear network effects enables a full-fledged dynamic analysis taking boundary solutions into account. The extent of network effects related to the separation rate and consumers' learning costs determines whether both types of software can coexist in the long run and whether a lock-in region emerges. Governmental promotion of free software increases welfare if network externalities are sufficiently large.

JEL-Classification: D43; D62; L13

Keywords: Software Market – Hotelling Model – Network Externalities – Lock-In

1 Introduction

There is ongoing competition between commercially and freely supplied software. Outstanding examples are MS-Windows™ versus Linux, MS-Office™ versus Open-Office, EViews™ versus gretl, and MS-Word™/Adobe-InDesign™ versus LaTeX. This paper deals with a dynamic model of asymmetric duopolistic competition between these two types of software suppliers taking network externalitites explicitly into account.

Free software comes with various types of licenses. Examples are *Public Domain*, the *GNU General Public License (GPL)*, and some mixed licenses. The GPL involves the idea of *Open Source*, that is, it guarantees that the source code of any software supplied under the GPL is freely available. However, it restricts the use of GPL source code by demanding that if this code becomes part of another software, the new software must also be published under the GPL. Thus, the notion of *free software* can have different meanings, depending on the effective type of license. The present paper simplifies matters by assuming related problems as e.g. the private appropriation of public ideas away. A software is denoted as *free* if it is provided free of charge. It will further be assumed that there are just two competing types of software, a commercially and a freely provided product.

The basic economics questions related to free software discussed in the literature are: Why are there suppliers of free software (Lerner and Tirole, 2002; Mustonen, 2003; Bitzer, 2004) and how do they respond to competition (Gaudeul, 2007)? How do commercial suppliers react in the presence of free competition (Mustonen, 2003; Casadesus-Masanell and Ghemawat, 2006; Bitzer, 2004; Lanzi, 2009)? Do stable equilibria exist (Casadesus-Masanell and Ghemawat, 2006)? What are the welfare implications of free software? Should the government encourage or even subsidize its development (Schmidt and Schnitzer, 2003; Mustonen, 2003; Comino and Manenti, 2005)? Of course, issues discussed in these papers are overlapping. The list indicates the respective focus of the cited papers.[1] The present paper is concerned with all but the first of the mentioned questions.

Among the important economic characteristics of software is, first, that production involves high fixed costs (due to R&D) but negligibly low variable costs of supplying another unit of an already developed product, implying nearly *joint consumability* on the demand side.[2] As Bitzer (2004, p. 379) puts it, "software production [is] a fixed-cost business resulting in a natural monopoly"; only with the emergence of open source software these formerly non-contestable markets have turned into oligopolies. Second, consumers' valuation of a specific software rises with the number of users and the availability of complementary software products. Thus, software involves *network externalities*.[3] Third, once a decision in favor of a particular product has been made it is costly to switch to another software, which could possibly create a *lock-in* effect, meaning that one choice becomes better than any other one just because everyone else already has made that choice.[4]

All of these mentioned characteristics play an important role in the present paper. Thus, due to joint consumability, the number of users will not affect the production costs of any software product. In contrast to the approaches of Schmidt and Schnitzer (2003) and Bitzer (2004) (who consider no externalities) and Comino and Manenti (2005) and Lanzi (2009) (who consider static or exogenous externalities), dynamic network externalities are explicitly taken into account. In the present model, no user who has made a decision will ever switch to the other product (he therefore *locks in* by definition).[5] The question arises, however, whether a lock in could occur in the sense that all new users choose the same software because of network externalities. Conditions for the emergence of such a lock-in region will explicitly be derived.

While the present approach is similar to that of Casadesus-Masanell and Ghemawat (2006) in some respects, it differs with respect to the modeling of the demand

[1] Important further sources on several aspects of free software are Hahn (2002), Schiff (2002), Fitzgerald and Basset (2003), and Gehring and Lutterbeck (2007).

[2] This characteristic of software clearly implies that pirated copies of software are a possible problem for commercial vendors. Cf. Shy and Thisse (1999) and Peitz (2004) for a discussion of software protection.

[3] While the seminal paper on network externalities by Katz and Shapiro (1985) is concerned with an analysis of private and social incentives to produce compatible products in a network, the network in the present model consists of the users of the same software. Gandal (1994) provides empirical evidence for the importance of network externalities in the computer spreadsheet industry.

[4] See Page (2006), e.g., for a discussion of the related concept of *path dependence* and an analysis of the most prominent example of a lock in, the QWERTY keyboard.

[5] Cf. Lanzi (2009) for an explicit analysis of lock-in effects due to the accumulation of experience that is not perfectly transferable between different types of software.

side. Instead of their specialized new approach, a standard Hotelling (1929) model of product differentiation will be employed here, as it has also been the case in the static models of Schmidt and Schnitzer (2003) or Comino and Manenti (2005). While Casadesus-Masanell and Ghemawat (2006) use optimal control theory to model the behavior of the commercial producer, who therefore takes the long-run development of network externalities into account, the present paper is concerned with a myopic producer. It is understood that real world vendors probably are forward-looking, but their real behavior will most likely lie somewhere in between the cases of perfect foresight and complete myopia. Thus, the assumption of a myopic producer is interesting in its own right as one of the limiting cases of real world behavior. Moreover, it simplifies matters considerably. It therefore allows for a more realistic modeling of other issues such as an explicit analysis of boundary solutions and the conditions for their emergence. The implications differ substantially: While the commercial software will never be pushed out of the market in Casadesus-Masanell and Ghemawat (2006), the outcome now depends on the extent of network externalities. If these are moderately positive, both types of software will coexist. If network effects become more important, the commercial software is completely pushed out of the market. Finally, in case of very high network-effects, the lock-in region already mentioned emerges and it becomes likely that the free software never can enter the market.

The Hotelling-model to be developed is related to the approaches of Schmidt and Schnitzer (2003) and Comino and Manenti (2005), who both focus on the welfare effects of public policy towards free software. While Schmidt and Schnitzer (2003) conclude that mandated adoption of free software always decreases social welfare, Comino and Manenti (2005) assert that this type of policy can increase welfare if there are uninformed consumers (who do not know of the existence of free software). Additional information provided by the government has a similar effect if network externalities are taken into account and will always increase welfare in the absence of externalities.[6] While the results of Schmidt and Schnitzer (2003) and Comino and Manenti (2005) mainly differ because of the presence of uninformed consumers in the latter paper, it will be shown that in the presence of dynamic network externalities mandated adoption can both decrease or increase social welfare, even if consumers have complete information.[7] Additionally, the present paper extends the existing literature by explicitly taking corner solutions into account where just one of the competitors stays in the market.

The model is presented in Section 2, starting with the analysis of the short-run equilibrium. Afterwards, the dynamics and the possible long-run equilibria are considered. The results are used in Section 3 to analyze the welfare implications of a free supplier's market entry and to evaluate government policy in favor of free software. The final section offers some concluding remarks.

[6] Mustonen (2003) has also a case for information policy.

[7] According to Comino and Manenti (2005, p. 218) most of the economics literature besides Varian and Shapiro (2007) is skeptical about government interventions in the software market.

2 A Model of Asymmetric Duopolistic Competition

2.1 Structure of the Model

The basic framework of the analysis is illustrated in Fig. 1. At each instant of time, the duopolists, that is the suppliers of the commercial and the free software, resp., make their short-run decisions. As the process of R&D will not be taken into account, these decisions pertain just to price setting. Thus, the strategy of the free supplier is determined by definition. He supplies its product free of charge, aiming at maximizing his market share. In contrast, the commercial supplier strives to set a profit maximizing price. These differing objectives of both duopolists explain why the model is concerned with *asymmetric* duopolistic competition.[8] At each instant of time, new potential customers of measure one enter the market. As a result of the commercial supplier's price setting behavior, the short-run market shares $\bar{x}$ and $1-\bar{x}$ are determined, where $\bar{x}$ is the share of new customers buying the commercial software. As software products are durable goods, it is assumed that any piece of software is used until the respective consumer retires.

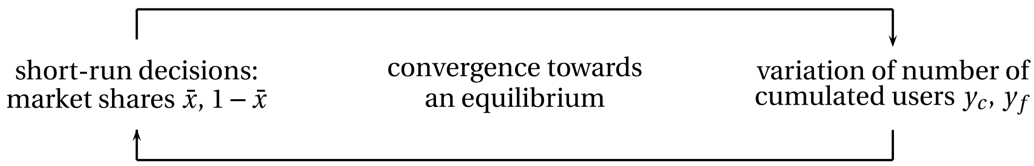


Figure 1. Static-Dynamic Interaction

The short-run market shares determine the variation of the numbers of cumulated users of the commercial and the free software, y_c and y_f, respectively.[9] Due to the existence of network externalities, these cumulated user numbers affect the new short-run equilibrium. It will be shown that this process of static-dynamic interaction always converges to a long-run equilibrium.

2.2 The Short-Run Equilibrium

Consider a standard Hotelling-like model of differentiated products. The commercial software is identified with quality 0, the free software with quality 1. Suppose that at any time t there is a continuum $[0,1]$ of uniformly distributed new potential customers identified by their position $x \in [0,1]$. Otherwise, all consumers are identical. The measure of all new customers is normalized to one.

Similar to the formulation in Schmidt and Schnitzer (2003), the money metric net utility of a consumer located at x from buying a unit of the commercial software is given by

$$U_c = V_c(y_c) - \tau x - p, \tag{1}$$

[8] One could argue that only the strategic decisions of one of the players, the commercial vendor, are explicitly analyzed. If there is another player whose objective is to maximize his market share, however, this objective plainly implies that his best strategy is to supply his software free of charge.

[9] While Comino and Manenti (2005) follow a static approach of Shy (2001) in which the number of users determined in a short-run equilibrium are at the same time a measure for the network externalities, the present paper presents a dynamic approach in which the network externalities depend on the cumulated number of users.

where $V_c(y_c) \geqq 0$ is money metric gross utility, y_c is the cumulated number of consumers using the commercial software, τx (where $\tau > 0$) represents the costs of learning how to use the new software of a new customer located at x, and p is the market price of the commercial software. As any consumer buys at most one unit of software and U_c is measured in monetary units, p in (1) should be interpreted as being multiplied by a "1" of dimension *software units*. Notice that $\tau x + p$ may be interpreted as the *total cost of ownership (TCO)* of the commercial software for an individual located at x. Net utility is therefore gross utility minus the TCO. As the TCO will in general not be zero even for a user of free software, it is possible that the commercial software is cheaper than the free software for some customers.

Net utility from using the free software is

$$U_f = V_f(y_f) - \tau(1 - x). \tag{2}$$

The nearer x is to 0 (to 1), the more likely a consumer prefers the commercial (the free) software.

The particular feature of the present model is the dependence of the money metric gross utility functions on the respective cumulated numbers of software users, y_c and y_f, which reflects the network externalities associated with software products. Fig. 2 illustrates the utility functions.

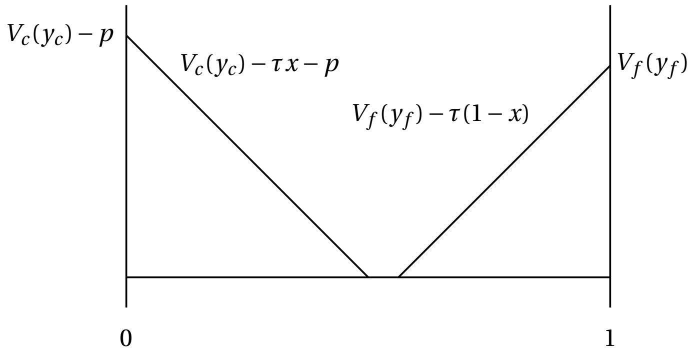


Figure 2. Utility as a Function of Distance

Let $V_i(y_i) \geqq 3\tau/4\ \forall y_i \in R_+$, where $R_+ := \{x \in R \mid x \geqq 0\}$, $i = c, f$. According to Proposition 1 below, this condition ensures that every consumer will use some software. Notice that the intersection of each function with the respective vertical axis in Fig. 2 increases with the respective number of cumulated users, y_i. Thus, if there are enough users, the two functions will intersect above the horizontal axis. The condition $V_i(y_i) \geqq 3\tau/4\ \forall y_i \in R_+$ ensures this. Let

$$V(y_c, y_f) := V_c(y_c) - V_f(y_f).$$

Then, equating (1) and (2) yields the indifferent consumer position $\bar{x}$ as

$$\bar{x} = \frac{1}{2} + \frac{V(y_c, y_f) - p}{2\tau}. \tag{3}$$

The solution to (3) is meaningful only if $\bar{x} \in [0, 1]$. Direct substitution of $V - \tau$ and $V + \tau$ for p shows that

$$\bar{x} \in [0, 1] \quad \Longleftrightarrow \quad V - \tau \leqq p \leqq V + \tau.$$

All consumers located at $x \in [0, \bar{x}]$ get higher utility from buying the commercial software, while those located at $x \in (\bar{x}, 1]$ will use the free software. Any consumer buys just one license, which he can use until he decides to use no software anymore. Thus, software is treated like a durable consumer good.

Turning to the supply side, the commercial producer strives to maximize his profit at any point in time. He knows the consumers' relevant demand function (3). As fixed costs are irrelevant with respect to marginal decisions and variable costs are negligibly small, it suffices to analyze just the problem of maximizing revenue $p\bar{x}$ by choosing the price p. This assumption considerably simplifies the analysis, although it should be kept in mind that the level of fixed costs will be important when deciding to stay in or to leave the market. Using (3) and $\bar{x} \in [0, 1]$, the commercial producer solves the following problem of quadratic programming at each instant of time:

$$
\begin{aligned}
& \max_{p} \quad p\bar{x} \\
\text{s.t.:} \qquad & \bar{x} = \frac{1}{2} + \frac{V(y_c, y_f) - p}{2\tau} \\
& p + \tau - V \geqq 0 \qquad (\Longleftrightarrow \ \bar{x} \leqq 1) \\
& V + \tau - p \geqq 0 \qquad (\Longleftrightarrow \ \bar{x} \geqq 0) \\
& p \geqq 0
\end{aligned}
\tag{4}
$$

The Kuhn-Tucker-conditions give rise to the following proposition describing the short-run equilibrium.

Proposition 1 *Let $V_i(y_i) \geqq 3\tau/4 \ \forall y_i \in R_+$, $i = c, f$. Then every consumer will use some software. Moreover:*

(a) If $V - 3\tau < 0 < V + \tau$, an interior solution $(p, \bar{x}) = (p_d, \bar{x}_d)$ applies:

$$
p = p_d := \frac{V + \tau}{2}, \qquad \bar{x} = \bar{x}_d := \frac{1}{4}\left(1 + \frac{V}{\tau}\right) \tag{5}
$$

(b) If $V + \tau \leqq 0$, no commercial software will be supplied and $p = 0$, $\bar{x} = 0$.

(c) If $V - 3\tau \geqq 0$, all consumers buy the commercial software, $\bar{x} = 1$, at the price $p = V - \tau$.

Proof: Appendix A. □

2.3 Dynamics and the Long-Run Equilibrium

At each point in time t there is a continuum of measure one representing new potential customers choosing a software product. Every customer determines which software product he will use until he retires. For the sake of simplicity, it is assumed that there is a constant separation rate δ indicating the percentage rate of the cumulated users who terminate using any software. Then the cumulated user numbers follow the differential equations

$$
\dot{y}_c = \bar{x} - \delta y_c, \quad \dot{y}_f = 1 - \bar{x} - \delta y_f. \tag{6}
$$

Since $\bar{x} \in [0,1]$, the minimum and maximum long-run user levels are obtained by setting $\dot{y}_c = 0$ and $\dot{y}_f = 0$ each for $\bar{x} = 0$ and $\bar{x} = 1$, respectively:

$$y_c^{\min} = 0, \quad y_f^{\min} = 0, \quad y_c^{\max} = \frac{1}{\delta}, \quad y_f^{\max} = \frac{1}{\delta}. \tag{7}$$

It will always be assumed that the initial values of y_i, $i = c, f$, do lie between these margins.

The fact that the long-run user levels are limited by the values given in (7) implies that it is reasonable to employ the simplest possible specification of gross utility functions, which are assumed to depend linearly on the respective user levels:

$$V_c(y_c) = a + b y_c, \quad V_f(y_f) = a + b y_f, \quad V(y_c, y_f) = b(y_c - y_f), \tag{8}$$

where a is a positive parameter representing the minimum gross utility created by each type of software and b is a positive parameter measuring the extent of network externalities. If the long-run user levels were not limited, intuition would suggest that the functions V_i are convex for small values of the user levels y_i and concave for high values. Given the limits in (7), the linear functions can be considered as a first order approximation to this more realistic form.

Applied to the linear specification (8), the assumption $V_i(y_i) \geqq 3\tau/4 \; \forall y_i \in R_+$, $i = c, f$, of Proposition 1 reads

$$a \geqq \frac{3}{4}\tau. \tag{9}$$

As the following analysis otherwise would get lost in the discussion of too many special cases, it will be assumed that (9) is generally met.

Using (7), the range of possible $V(y_c, y_f)$-values follows from (8) as

$$V(y_c, y_f) \in \left(-\frac{b}{\delta}, \frac{b}{\delta}\right).$$

Suppose that $0 < b < \delta\tau$, implying that $b/\delta < \tau$ and $-b/\delta > -\tau$ and therefore that $-\tau < V < \tau$. According to Proposition 1, an interior solution applies if $-\tau < V < 3\tau$. Thus, the condition $0 < b < \delta\tau$ is sufficient for an interior solution to apply for all time. Setting

$$\epsilon := \frac{b}{\delta\tau} > 0,$$

this condition can be rewritten as $0 < \epsilon < 1$. The parameter ϵ measures the extent of network effects in relation to the product of the separation rate and the rate of learning costs. It therefore may be interpreted as measuring the relative importance of network externalities. The case $\epsilon = 0$ would imply that there were no network externalities and will therefore not be considered.

At first, assume that $0 < \epsilon < 1$ (that is, $0 < b < \delta\tau$). Substituting $\bar{x}$ from (5) into (6), taking (8) into account and using $\epsilon\delta\tau$ instead of b yields the linear second order system of differential equations

$$\begin{pmatrix} \dot{y}_c \\ \dot{y}_f \end{pmatrix} = \begin{pmatrix} -\left(1 - \frac{1}{4}\epsilon\right)\delta & -\frac{1}{4}\epsilon\delta \\ -\frac{1}{4}\epsilon\delta & -\left(1 - \frac{1}{4}\epsilon\right)\delta \end{pmatrix} \begin{pmatrix} y_c \\ y_f \end{pmatrix} + \begin{pmatrix} \frac{1}{4} \\ \frac{3}{4} \end{pmatrix} \tag{10}$$

Denote the coefficient matrix by J. Then, $\mathrm{Tr}(J) = -(2-\epsilon/2)\delta < 0$ and $|J| = (2-\epsilon)\delta^2/2 > 0$. Thus, the Routh-Hurwitz conditions are satisfied for this linear system, which is necessary and sufficient for global asymptotic stability of the long-run equilibrium values in case of duopoly. These are (y_{cd}, y_{fd}), obtained by setting $\dot{y}_c = \dot{y}_f = 0$ and solving (10) for y_c and y_f:

$$y_c = y_{cd} := \frac{1-\epsilon}{2\delta(2-\epsilon)}, \quad y_f = y_{fd} := \frac{3-\epsilon}{2\delta(2-\epsilon)}. \tag{11}$$

The long-run market share of the free software exceeds the market share of the commercial software. In summary:

Proposition 2 *If $0 < b < \delta\tau$ ($0 < \epsilon < 1$, resp.), the long-run equilibrium is globally asymptotically stable. Both types of software have a positive market share with user levels (11) in the long run.*

As to the empirical application of this proposition it should be kept in mind that actually the market shares of commercial software products may be larger than indicated by (11) because of the presence of uninformed consumers stressed by Comino and Manenti (2005). These consumers are simply neglected in the present model.

Next suppose that $\epsilon \geqq 1$. Although there is nothing to assure now that $V - 3\tau < 0 < V + \tau$ in the long run, $\epsilon \geqq 1$ does by itself neither preclude interior short-run solutions nor does it imply that the condition $V - 3\tau < 0 < V + \tau$ is actually violated in the long run. As $\bar{x} \in (0,1)$ as long as $V - 3\tau < 0 < V + \tau$, inspection of equations (11) and the coefficient matrix J yields some information about the possible long-run equilibria in case of $\epsilon \geqq 1$. From (11), if $\epsilon = 1$, y_c converges to zero and the free software takes over the complete market (convergence is proven below). Next, if $1 < \epsilon \leqq 3$, a positive long-run equilibrium solution according to (11) does not exist. Thus, it is impossible for both competitors to survive in the long run. Finally, if $\epsilon > 3$, $|J| = (2-\epsilon)\delta^2/2 < 0$, implying that any interior long-run equilibrium is an unstable saddle point.

In light of these results, $\epsilon \geqq 1$ implies that boundary solutions where $\bar{x} = 0$ or $\bar{x} = 1$ have to be considered from some point in time on. The differential equations describing the long-run behavior must be modified accordingly. Although analytical solutions are feasible, the various possible cases are most easily discussed using phase diagrams. Fig. 3 illustrates the principle construction of these diagrams.

Between the dotted lines $V = -\tau$ and $V = 3\tau$, both types of software are supplied, since $\bar{x} = (1 + V/\tau)/4 \in (0,1)$. Using (8) and $b = \epsilon\delta\tau$, these lines are given by $y_f = y_c + 1/(\epsilon\delta)$ and $y_f = y_c - 3/(\epsilon\delta)$, respectively. Above $V = -\tau$, $\bar{x} = 0$, and below $V = 3\tau$, $\bar{x} = 1$.

From (6) and (10), the isocline $\dot{y}_c = 0$ is given by

$$\begin{aligned} &y_c = 0 && \text{if} \quad \bar{x} = 0 \\ &y_f = \frac{1}{\epsilon\delta} - \left(\frac{4-\epsilon}{\epsilon}\right) y_c && \text{if} \quad \bar{x} \in (0,1) \\ &y_c = 1/\delta && \text{if} \quad \bar{x} = 1 \end{aligned}$$

The interior part of the isocline thus always has the same intercept with the y_f-axis as the line $V = -\tau$. It is negatively sloped as long as $\epsilon \in (0,4)$, and its interception

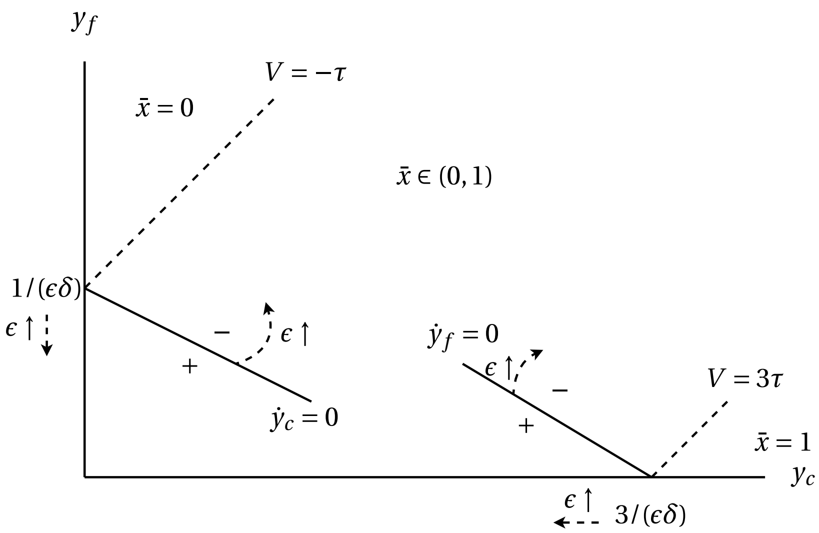


Figure 3. Construction of the Phase Diagrams

with the $V = 3\tau$ line can be calculated to be at $(y_c, y_f) = (1/\delta, (\epsilon-3)/(\epsilon\delta))$, which lies in the positive orthant only if $\epsilon > 3$ (as in Fig. 6 below). Thus, the upper part of the isocline looks qualitatively as shown in Fig. 3, where the lower part depends on the numerical value of ϵ. As the parameter ϵ is increased, the isocline will rotate counterclockwise around its intercept with the y_f-axis (which is moving towards the origin). The movement of the isocline $\dot{y}_c = 0$ as ϵ increases is indicated by the dotted arrows near this isocline in Fig. 3.

Similarly, from (6) and (10), the isocline $\dot{y}_f = 0$ is given by

$$
\begin{aligned}
&y_f = 1/\delta && \text{if} \quad \bar{x} = 0 \\
&y_c = \frac{3}{\epsilon\delta} - \left(\frac{4-\epsilon}{\epsilon}\right) y_f && \text{if} \quad \bar{x} \in (0,1) \\
&y_f = 0 && \text{if} \quad \bar{x} = 1
\end{aligned}
$$

The interior part of the isocline thus always has the same intercept with the y_c-axis as the line $V = 3\tau$. Its interception with the $V = -\tau$ line can be calculated to be at $(y_c, y_f) = ((\epsilon-1)/(\epsilon\delta), 1/\delta)$, which lies in the positive orthant only if $\epsilon > 1$ (as in Figures 5 and 6 below). Thus, the lower part of the isocline looks qualitatively as shown in Fig. 3. As the parameter ϵ is increased, the isocline will rotate clockwise around its intercept with the y_c-axis (which is moving towards the origin). Again, the movement of the isocline $\dot{y}_f = 0$ as ϵ increases is indicated by the dotted arrows near this isocline in Fig. 3.

This discussion of slopes and intercepts shows that the critical values of ϵ, where the dynamic behavior of solutions might radically change, are 0, 1, 3, and 4. The directions of movement are indicated by the + and − signs in Fig. 3, which follow from the partial derivatives of $\dot{y}_c$ and $\dot{y}_f$ with respect to y_f and y_c, respectively, which both are $-\epsilon\delta/4 < 0$.

Fig. 4 shows the case where $0 < \epsilon < 1$, already discussed in Proposition 2, and proves the former assertion that y_c converges to zero if $\epsilon = 1$. Notice that initial values near $(y_c, y_f) = (1/\delta, 0)$ are plausible in that they would arise if the user level of the commercial software was already near its long-run equilibrium value when the free software entered the market. The Figure shows that the free software indeed could enter the market and possibly even supersede the commercial software in such a case.

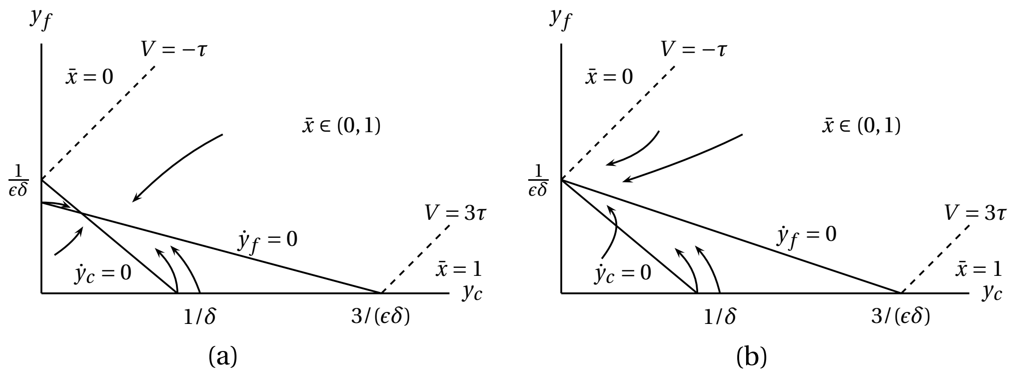


Figure 4. (a) Stable Interior Solution for $0 < \epsilon < 1$, (b) Stable Corner Solution for $\epsilon = 1$

Fig. 5 shows the cases where $1 < \epsilon < 3$ and $\epsilon = 3$, respectively. The interior equilibrium disappears and the free software will take over the complete market. Only if $\epsilon = 3$ and the initial values satisfy $(y_c, y_f) = (1/\delta, 0)$, there is an unstable equilibrium where the commercial software could stay in the market. If a few pioneers nevertheless began to use the free software, however, due to the instability of $(1/\delta, 0)$ the commercial software would again be pushed out of the market.

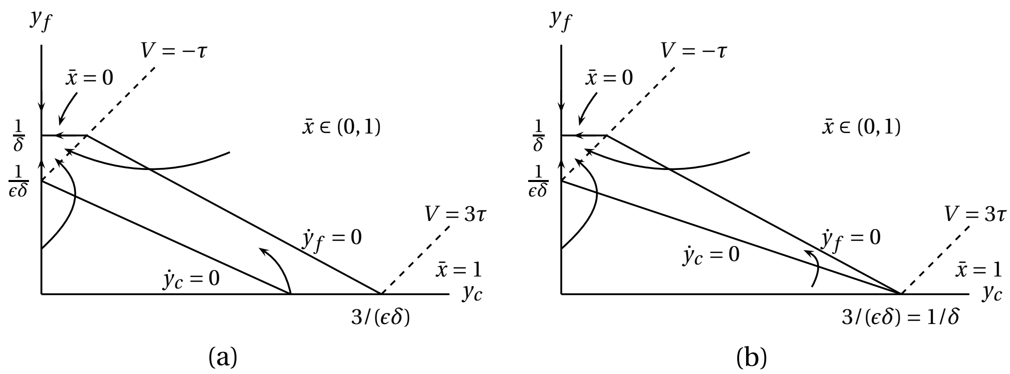
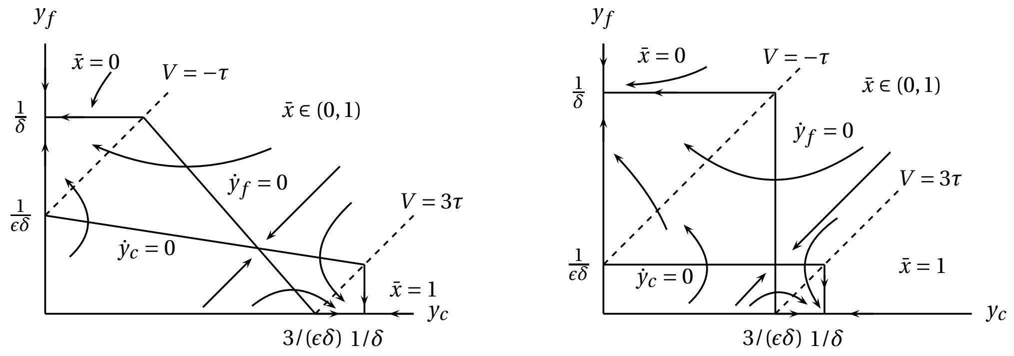


Figure 5. (a) Stable Corner Solution for $1 < \epsilon < 3$, (b) Stable Corner Solution for $\epsilon = 3$ and Unstable Equilibrium at $(y_c, y_f) = (1/\delta, 0)$

Finally, in Fig. 6, where $3 < \epsilon < 4$ and $\epsilon = 4$, a new interior saddle point-equilibrium emerges. The stable arm of the saddle point separates the regions that become attractive for the two stable corner equilibria where only one of the producers survives. In other words, to the right of the stable arm lies a lock-in region. If both types of software entered the market at the same time with the same number of initial users, the free software would eventually supersede the commercial software. If the commercial software already has an advantage such that the initial values lie to the right of the stable arm of the interior saddle point, however, the free software cannot enter the market because the network externalities associated with a high number of commercial software users would have created the lock-in region comprising all initial values to the right of the stable saddle path. The case where $\epsilon > 4$ has not been depicted as no additional qualitative insights would have been gained. The slopes of both isoclines become positive but remain smaller than one in this case, implying that the interior equilibrium does not vanish and continues to be a saddle point.



(a) (b)

Figure 6. (a) Saddle Point for $3 < \epsilon < 4$ with Lock-In Region, (b) Saddle Point for $\epsilon = 4$

The main results are summarized in

Proposition 3 *(a) If $1 \leqq \epsilon < 3$, the free software supplier enters the market and supersedes the commercial supplier.*

(b) If $\epsilon = 3$, the commercial software will be superseded unless the initial values are $y_f(0) = 0$ and $y_c(0) = 1/\delta$ (where $\bar{x} = 1$).[10]

*(c) If $\epsilon > 3$, there is a region of initial values determined by the stable arm of an interior saddle point such that $(y_c, y_f) = (1/\delta, 0)$ is a stable equilibrium. Thus, the free software cannot enter the market if the initial point lies to the right of the stable arm of the saddle point (*lock-in effect*).*

These results differ substantially from those of Casadesus-Masanell and Ghemawat (2006), where the commercial software will never be pushed out of the market. The extent of network externalities in Casadesus-Masanell and Ghemawat (2006) depends on a parameter s. In their model, either both types of software coexist in the long run (if $s > 1$) or the commercial software supersedes the free software (if $s \leqq 1$). The effect of the parameter ϵ in the present model is considerably more complicated. In particular, the effect of network externalities is not monotonous. If $0 < \epsilon < 1$, the long-run market share of the free software increases as ϵ rises (notice that $\partial y_f / \partial \epsilon > 0$ and $\partial y_c / \partial \epsilon < 0$ according to equations (11)). If $1 \leqq \epsilon < 3$, the commercial software is completely pushed out of the market. If ϵ rises further, however, the lock-in region emerges and it becomes likely that the free software never can enter the market.

[10] It would suffice if the initial point was lying to the right of the line $V = 3\tau$. Since $y_c(0) > 1/\delta$ in this case, however, such an initial point does not make much sense.

3 Welfare and Policy Implications

3.1 Market Entry of a Free Supplier

There are two major questions related to welfare effects and economic policy:[11]

1. Does social welfare as measured by the sum of producer's and consumer's surplus increase as a result of a free software supplier's market entry? Otherwise, governmental promotion of such an entry could not be beneficial.

2. Could the government increase social welfare by compelling some consumers (public agencies, e.g.) to use the free software (mandated adoption policy)?

These questions will be answered with respect to the long-run equilibrium of the model.

As a first step of welfare considerations, the market solution in case of a pure monopoly must be determined. Using (1), the producer's optimization problem reads:

$$\begin{aligned} &\max_{p_m} \quad p_m \bar{x}_m \\ \text{s.t.:} \qquad &\bar{x}_m = \frac{V_c(y_c) - p_m}{\tau} \quad \text{(from setting } U_c = 0) \\ &p_m + \tau - V_c \geqq 0 \qquad (\Longleftrightarrow \bar{x}_m \leqq 1) \\ &V_c - p_m \geqq 0 \qquad (\Longleftrightarrow \bar{x}_m \geqq 0) \\ &p_m \geqq 0, \end{aligned} \tag{12}$$

where p_m and $\bar{x}_m$ are the monopoly price and the marginal consumer on $[0, 1]$ buying the commercial software. The implications of this problem are summarized in[12]

Proposition 4 *(a) If $0 < V_c < 2\tau$, an interior solution applies:*

$$p_m = \frac{V_c}{2}, \qquad \bar{x}_m = \frac{V_c}{2\tau} \tag{13}$$

(b) If $V_c \leqq 0$, no software will be supplied and $p_m = 0$, $\bar{x}_m = 0$.

(c) If $V_c \geqq 2\tau$, all consumers buy the commercial software, $\bar{x}_m = 1$, at the price $p_m = V_c - \tau$.

Proof: Analogous to the proof of Proposition 1. □

Using (13) and the specifications in (6) and (8), the implied differential equation in case of an interior solution is

$$\dot{y}_c = \frac{a}{2\tau} - \left(1 - \frac{\epsilon}{2}\right) \delta y_c, \tag{14}$$

[11] Governments could also prefer open source software to commercial software because they keep the freedom to change the source code according to their needs, cf. e.g. Varian and Shapiro (2007). This issue is out of the scope of the present model, however.

[12] The literature on asymmetric competition in the software market has no definite answer on whether a former monopolist will raise or reduce its price when a free competitor enters the market, cf. e.g. Lanzi (2009). A comparison of (13) with (5) and (16) with (18) regarding the short and the long run, respectively, shows that in the present model neither case can be ruled out too.

where $\epsilon = b/(\delta\tau)$ as before. As (7) applies analogously, $V_c \in (a, a + b/\delta)$. A sufficient condition for an interior solution is therefore given by

$$a > 0 \quad (\Rightarrow V_c > 0) \quad \text{and} \quad a < (2-\epsilon)\tau \quad (\Rightarrow V_c < 2\tau).$$

In order to be specific, the parameters shall meet the conditions

$$(2-\epsilon)\tau > a \geqq \frac{3}{4}\tau > 0, \quad \text{and} \quad 0 < \epsilon \leqq 1, \tag{15}$$

ensuring an interior solution in case of monopoly and repeating assumptions of the duopoly model. Using (8), (13), and (14) solved for y_c by setting $\dot{y}_c = 0$, and $b = \epsilon\delta\tau$, the globally asymptotically stable long-run interior monopoly equilibrium is thus described by:

$$y_c = y_{cm} := \frac{a}{\delta\tau(2-\epsilon)}, \quad V_c(y_{cm}) = \frac{2a}{2-\epsilon}, \quad p_m = \frac{a}{2-\epsilon}, \quad \bar{x}_m = \frac{a}{\tau(2-\epsilon)}. \tag{16}$$

The sum of consumer's and producer's surplus in long-run equilibrium can now be calculated. In case of the pure monopoly, using (16) one gets

$$W_M := \int_0^{\bar{x}_m} (V_c(y_{cm}) - \tau x - p_m)dx + p_m\bar{x}_m = \frac{3a^2}{2\tau(2-\epsilon)^2}. \tag{17}$$

In case of asymmetric duopolistic competition, inserting (11) into (8) and (5), respectively, yields

$$V_c(y_{cd}) = a + \frac{\epsilon\tau(1-\epsilon)}{2(2-\epsilon)}, \; V_f(y_{fd}) = a + \frac{\epsilon\tau(3-\epsilon)}{2(2-\epsilon)}, \; p_d = \tau\frac{1-\epsilon}{2-\epsilon}, \; \bar{x}_d = \frac{1-\epsilon}{2(2-\epsilon)} \tag{18}$$

A tedious but straightforward calculation (cf. Appendix B) then shows that the sum of surpluses in case of duopoly is

$$\begin{aligned} W_D &:= \int_0^{\bar{x}_d} (V_c(y_{cd}) - \tau x - p_d)dx + \int_{\bar{x}_d}^1 (V_f(y_{fd}) - \tau(1-x))dx + p_d\bar{x}_d \\ &= \frac{2a(2-\epsilon)^2 + \tau\left(\epsilon^3 - 4.5\epsilon^2 + 7\epsilon - 2.5\right)}{2(2-\epsilon)^2}. \end{aligned}$$

It is also shown in Appendix B that the difference $W_D - W_M$ is positive, zero or negative according to whether the ratio of minimum gross utility a to learning costs τ meets

$$\left(\frac{a}{\tau}\right)^2 - \frac{2(2-\epsilon)^2}{3}\frac{a}{\tau} - \frac{\epsilon^3 - 4.5\epsilon^2 + 7\epsilon - 2.5}{3} \lesseqqgtr 0.$$

Even if the assumptions (15) are met, all cases are possible. E.g., if $a/\tau = 3/4$ and $\epsilon = 1$, or if $a = \tau$ and $0 < \epsilon \leqq 1$, the preceeding expression is negative and thus $W_D - W_M > 0$. If $\tau = 1$, $\epsilon = 0.8$, and $a = 1.19$, however, then $W_D - W_M < 0$. As shown in Appendix B, $W_D - W_M < 0$ is possible only if $0.5 < \epsilon < 1$ and a comes close to its upper bound specified in (15). More precisely, as a/τ must lie between the two gray lines in Fig. 7 because of the assumptions (15), and since $W_D > W_M$ between the two black curves, the only region where $W_D < W_M$ is the small lens limited by the upper black curve $W_D = W_M$ and the straight line $2-\epsilon$ in Fig. 7 between $\epsilon = 0.5$ and $\epsilon = 1$. Thus, for a

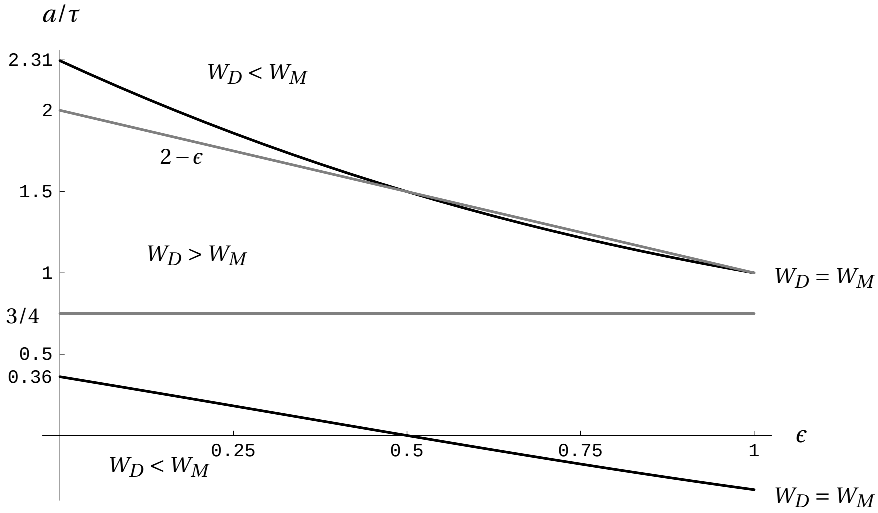


Figure 7. Parameter Regions of Welfare Effects of Market Entrance

very wide range of parameter values, the market entry of the free supplier will raise social welfare.

For an economic explanation of these results, observe that the assumptions (15) ensure an interior solution under monopoly and, as long as $\epsilon < 1$, under duopoly. When the free software enters the market, new consumers who formerly would not have used any software will use the free software, which increases welfare. If the number of new users of the commercial software is not reduced, no negative effect on welfare exists. Thus, the gross effect on welfare is unequivocally positive. If the commercial software looses new customers, however, there is a loss due to decreased profits of the commercial supplier and additionally due to a decreased extent of network externalities associated with the commercial software, deteriorating gross utility of all remaining users of the commercial software. This effect can outweigh the positive effect on new users of the free software, but only for a very limtited range of parameter values as shown in Fig. 7. The smaller a is in relation to τ, the smaller is the number of users of the commercial software under monopoly and the smaller is their (possible loss of) net utility. Thus, a relatively high value of a/τ is necessary for a negative welfare effect of market entrance of the free supplier.

3.2 Governmental Regulation in Favor of Free Software

When the free software already has entered the market, the government could contemplate whether compelling a fraction of customers that otherwise would choose the commercial software to choose the free software could increase social welfare (mandated adoption policy). Although compelling a fraction of *all* consumers may appear somewhat artificial, proceeding this way turns out to be a straightforward generalization of the preceding analysis. Moreover, it is possible to directly compare the results with those of Schmidt and Schnitzer (2003) and Comino and Manenti (2005). The implied assumption is that government employees are uniformly distributed over $[0,1]$ like the rest of the population. Compelling all or a fraction of governmental employees to use the free software then amounts to compelling some other fraction η of all customers.

Setting up the commercial producer's optimization problem shows that analytically such a policy leaves the determination of $\bar{x}$ and p in static equilibrium of the asymmetric duopoly unaffected. The reason is that from the point of view of the commercial producer this policy amounts to nothing more than multiplying his objective function in (4) by the constant $(1-\eta)$, which does not influence the optimum decision as long as the profit remains non-negative. The values p_d and $\bar{x}_d$ from (5) are therefore valid as before. The number of new customers to be used in the differential equations, however, changes from $\bar{x}$ to $(1-\eta)\bar{x}$ for $\dot{y}_c$ and from $1-\bar{x}$ to $1-(1-\eta)\bar{x}$ for $\dot{y}_f$, respectively. This gives rise to the new long-run equilibrium values

$$y_c = y_{c\eta} := \frac{(1-\epsilon)(1-\eta)}{2\delta(2-(1-\eta)\epsilon)}, \quad y_f = y_{f\eta} := \frac{3-(1-\eta)\epsilon+\eta}{2\delta(2-(1-\eta)\epsilon)}.$$

Depending on y_c and y_f, the short-run equilibrium values $\bar{x}$ and p are also affected. As the government policy under consideration requires that both types of software actually are supplied, it will be assumed that $0 < \epsilon < 1$.

Calculating the sum of consumer's and producer's surplus is straightforward but rather tedious if done by paper and pencil, however. A Mathematica™-file containing these calculations is therefore available from the author upon request. The result for the sum of consumer's and producer's surplus is given by

$$\begin{aligned}
&(1-\eta)\left[\int_0^{\bar{x}_d}(V_c(y_{c\eta})-\tau x-p_d)dx+\int_{\bar{x}_d}^1(V_f(y_{f\eta})-\tau(1-x))dx\right]\\
&\qquad +(1-\eta)p_d\bar{x}_d+\eta\int_0^1(V_f(y_{f\eta})-\tau(1-x))dx\\
&= \frac{2a\tau[2+\epsilon(\eta-1)]^2+[\epsilon^3(\eta-1)^2+4.5\epsilon^2(\eta-1)+\epsilon(7+\eta)-1.5\eta-2.5]\tau^2}{2\tau[2+\epsilon(\eta-1)]^2}
\end{aligned}$$

Observe that this expression equals W_D if $\eta = 0$. Calculating the derivative with respect to η yields

$$\frac{(1-\epsilon)(\epsilon^2(\eta-1)+\epsilon(11+3\eta)-6)\tau}{4(2-(1-\eta)\epsilon)^3},$$

which is positive if

$$\epsilon(11+3\eta) > 6+\epsilon^2(1-\eta).$$

In particular, if $\eta = 0$, the derivative is positive whenever $\epsilon > 0.576$. More generally, compelling customers to use the free software increases social welfare if there is a relatively large extent of network externalities ($0 < \epsilon < 1$ near one). E.g., even if $\eta = 0.5$ and $0.5 < \epsilon < 1$, the inequality is met. This effect is not present in the models of Comino and Manenti (2005) and Schmidt and Schnitzer (2003). The latter find that governmental regulation is generally detrimental. The results differ because of the dynamic network externalities taken into account here. It must be noted, however, that this kind of governmental regulation will be harmful even in the present model if the extent of network externalities is rather limited.

Economically, a positive effect of regulation in favor of free software may be explained by the fact that even before regulation a large fraction of consumers are using the free software and these consumers benefit from additional network externalities generated by additional government users. These benefits can outweigh the losses of those employees that are forced to use the free software and the remaining users who suffer from decreased network externalities generated for the commercial software.

As an additional remark, it should be noted that employees of the public sector do not have to bury the costs for the software they use at work. Their preferences may therefore be distorted in that they regard both types of software as *free*, although the government has to raise taxes in order to finance licenses for the commercial software. There may arise an additional benefit from using free software because tax distortions could be reduced. (Notice that it is not possible to add the saved government expenditures as such to the benefits. What the government saves is just what the commercial producer looses.)

4 Concluding Remarks

This paper analyzes the welfare effects of free software in asymmetric duopolistic competition taking boundary solutions and the dynamics of network externalities fully into account. The results of the model are only valid so far, however, if producers are myopic in that they do not take the long-run network externalities into account.

Among the main results is that both types of software can coexist in the long run and that the solution always converges to an equilibrium. With increasing importance of network externalities the free software gets a larger market share, except for very high externalities that are likely to produce a lock-in region preventing the free software from entering the market. If there is competition, governmental promotion of free software can be welfare increasing due to dynamic network externalities.

At the other extreme, producers would have perfect foresight and solve a problem of optimal control induced by the network externalities. Having to rely on suppositions at the moment, the main effect of this alternative assumption would be that the commercial supplier probably chose a lower price in short-run equilibrium taking the externalities into account, putting him into a slightly better long-run position. On the other hand, fixed costs of production have been completely neglected. Although these fixed costs do not affect marginal decisions, they can be decisive with respect to the decision of staying in or leaving the market. Thus, if fixed costs were taken into account, the position of the commercial supplier would deteriorate. A more thorough analysis of these issues is left for future research.

Appendix A

Proof of Proposition 1

The proposition is proven by deriving the optimum solution for the three cases referred to. In each case it is also necessary to show that the solution indeed always implies that all consumers use some software. The Lagrangean of problem (4) is

$$L = p\frac{1}{2} + p\frac{V(y_c, y_f) - p}{2\tau} + \lambda_1(p + \tau - V(y_c, y_f)) + \lambda_2(V(y_c, y_f) + \tau - p).$$

As (4) is a problem of quadratic programming, the following Kuhn-Tucker conditions are necessary and sufficient for a maximum:

$$L_p = \frac{1}{2} + \frac{V-2p}{2\tau} + \lambda_1 - \lambda_2 \leqq 0, \qquad p \geqq 0, \qquad L_p p = 0, \tag{A.1}$$

$$L_{\lambda_1} = p + \tau - V \geqq 0, \qquad \lambda_1 \geqq 0, \qquad L_{\lambda_1}\lambda_1 = 0, \tag{A.2}$$

$$L_{\lambda_2} = V + \tau - p \geqq 0, \qquad \lambda_2 \geqq 0, \qquad L_{\lambda_2}\lambda_2 = 0. \tag{A.3}$$

Beginning with case (b) suppose that $V+\tau = 0$. From (A.3) and $p \geqq 0$, this implies $p = 0$. Substitution into (3) yields $\bar{x} = 0$. If $V+\tau < 0$, there is no admissible solution to problem (4), and $\bar{x} = 0$ is the boundary solution following from (3). To prove that all consumers use software, observe that $V \leqq -\tau$ implies $V_f \geqq V_c + \tau \geqq 3\tau/4 + \tau$ because of the assumption that $V_i(y_i) \geqq 3\tau/4\ \forall y_i \in R_+,\ i = c, f$. Substituting $3\tau/4+\tau$ for V_f into U_f shows that $U_f \geqq 3\tau/4 > 0$, implying that all consumers use the free software. This proves part (b) of the proposition.

Next let $V - 3\tau \geqq 0$ as in (c). Since this implies $V - \tau > 0$, it follows from (A.2) that $p > 0$ and therefore $L_p = 0$ according to (A.1). If $p > V - \tau$, (A.2) implies $\lambda_1 = 0$, and (A.1) yields

$$p = \frac{V+\tau}{2} - \lambda_2\tau,$$

which together with the assumption $p > V - \tau$ implies

$$V < 3\tau - 2\lambda_2\tau,$$

a contradiction to $V - 3\tau \geqq 0$. Thus, $p = V - \tau$ according to (A.2), implying $\lambda_2 = 0$ from (A.3). Substituting for p in (A.1) implies $V = 3\tau + 2\lambda_1\tau$ (consistent with $V - 3\tau \geqq 0$). Substituting $p = V - \tau$ into (3) yields $\bar{x} = 1$. As substitution of $p = V - \tau$ and $\bar{x} = 1$ into U_c yields $U_c = V_f \geqq 3\tau/4 > 0$, all consumers will use the commercial software. This proves part (c).

Finally, let $V - 3\tau < 0 < V + \tau$. Suppose that $p = V + \tau$ and therefore $\lambda_1 = 0$ from (A.2). Upon substitution into (A.1) one gets $V + \tau = -2\tau\lambda_2 \leqq 0$, contradicting the assumption. Next, suppose that $p = V - \tau$ and therefore $\lambda_2 = 0$ from (A.3). Substitution into (A.1) now yields $V = 3\tau + 2\lambda_1\tau \geqq 3\tau$, again contradicting the assumption. Thus, it follows that $V - \tau < p < V + \tau$ and thus $\lambda_1 = \lambda_2 = 0$ from (A.2) and (A.3), which by substitution into (A.1) shows that $p > 0$ (since $0.5 + V/(2\tau) > 0$ if $p = 0$ and $V > -\tau$) and therefore implies from $L_p = 0$ that

$$p = \frac{V+\tau}{2}.$$

Substitution into (3) yields $\bar{x} = (1 + V/\tau)/4$. Finally, substituting these values for $\bar{x}$ and p into (1) and (2) yields

$$U_c(y_c) \geqq 0 \iff V_c + 3V_f \geqq 3\tau,$$
$$U_f(y_f) \geqq 0 \iff V_c + 3V_f \geqq 3\tau.$$

The assumption $V_i(y_i) \geqq 3\tau/4\ \forall y_i \in R_+,\ i = c, f$, implies that the inequality $V_c + 3V_f \geqq 3\tau$ is met. Thus, every consumer will have non-negative net utility and use a software product, proving (a).

Appendix B

Derivation of Fig. 7

The sum of consumer's and producer's surplus in long-run equilibrium of the asymmetric duopoly is

$$\begin{aligned} W_D &:= \int_0^{\bar{x}_d} (V_c(y_{cd}) - \tau x - p_d) dx + \int_{\bar{x}_d}^1 (V_f(y_{fd}) - \tau(1-x)) dx + p_d \bar{x}_d \\ &= V_c(y_{cd})\bar{x}_d - \frac{1}{2}\tau\bar{x}_d^2 - p_d\bar{x}_d + V_f(y_{fd}) - \frac{1}{2}\tau + [\tau - V_f(y_{fd})]\bar{x}_d - \frac{1}{2}\tau\bar{x}_d^2 + p_d\bar{x}_d \\ &= V_f(y_{fd}) + [V_c(y_{cd}) - V_f(y_{fd}) + \tau]\bar{x}_d - \tau\bar{x}_d^2 - \frac{1}{2}\tau \\ &= \frac{2a(2-\epsilon)^2 + \tau\left(\epsilon^3 - 4.5\epsilon^2 + 7\epsilon - 2.5\right)}{2(2-\epsilon)^2}, \end{aligned}$$

where the values provided in (18) have been substituted. Subtracting W_M given in (17) from W_D shows that $W_D - W_M \gtreqqless 0$ if

$$2a\tau(2-\epsilon)^2 + \left(\epsilon^3 - 4.5\epsilon^2 + 7\epsilon - 2.5\right)\tau^2 - 3a^2 \gtreqqless 0.$$

Dividing by τ^2 and rearranging, one finally gets

$$W_D - W_M \gtreqqless 0 \quad \Longleftrightarrow \quad \left(\frac{a}{\tau}\right)^2 - \frac{2(2-\epsilon)^2}{3}\frac{a}{\tau} - \frac{\epsilon^3 - 4.5\epsilon^2 + 7\epsilon - 2.5}{3} \lesseqqgtr 0.$$

This is a quadratic equation in a/τ, the solutions of which are easily calculated but are relatively complicated functions of ϵ. Call them $z_1(\epsilon)$ and $z_2(\epsilon)$, respectively, and choose them such that $z_1(\epsilon) \leqq z_2(\epsilon)$. If $z_1(\epsilon) < a/\tau < z_2(\epsilon)$, the quadratic expression is negative and therefore $W_D - W_M > 0$. Now observe that (15) requires that $3/4 \leqq a/\tau < 2-\epsilon$ and $0 < \epsilon \leqq 1$. In order to determine whether $W_D - W_M$ is positive, it is therefore possible to use a software solution to draw $z_1(\epsilon)$ and $z_2(\epsilon)$ (where in both cases $W_D = W_M$) and 3/4 and $2-\epsilon$ against ϵ for $0 < \epsilon \leqq 1$. The result is Fig. 7. Assumption (15) ensures that a/τ is between the lines 3/4 and $2-\epsilon$. Fig. 7 shows that most probably a/τ is thus also between $z_1(\epsilon)$ and $z_2(\epsilon)$, implying that $W_D > W_M$. Notice that $z_2(\epsilon)$ intersects $2-\epsilon$ at $\epsilon = 0.5$, implying that $W_D - W_M < 0$ is impossible if $\epsilon \leqq 0.5$.

References

Bitzer, J. (2004): Commercial versus Open Source Software: The Role of Product Heterogeneity in Competition, *Economic Systems*, 28, 369–381.

Casadesus-Masanell, R. and Ghemawat, P. (2006): Dynamic Mixed Duopoly: A Model Motivated by Linux vs. Windows, *Management Science*, 52, 1072–1084.

Comino, S. and Manenti, F. M. (2005): Government Policies Supporting Open Source Software for the Mass Market, *Review of Industrial Organization*, 26, 217–240.

Fitzgerald, B. and Basset, G. (Eds.) (2003): *Legal Issues Relating to Free and Open Source Software*, Queensland: Queensland University of Technology.

Gandal, N. (1994): Hedonic Price Indexes for Spreadsheets and an Empirical Test for Network Externalities, *RAND Journal of Economics*, 25, 160–170.

Gaudeul, A. (2007): Do Open Source Developers Respond to Competition? The (La)TeX Case Study, *Review of Network Economics*, 6, 239–263.

Gehring, R. A. and Lutterbeck, B. (Eds.) (2007): *Open Source Jahrbuch 2007 – Zwischen freier Software und Gesellschaftsmodell*, Berlin: Lehmanns Media.

Hahn, R. W. (Ed.) (2002): *Government Policy toward Open Source Software*, Washington, D.C.: Brookings Institution Press.

Hotelling, H. (1929): Stability in Competition, *Economic Journal*, 39, 41–57.

Katz, M. L. and Shapiro, C. (1985): Network Externalities, Competition, and Compatibility, *American Economic Review*, 75, 424–440.

Lanzi, D. (2009): Competition and Open Source with Perfect Software Compatibility, *Information Economics and Policy*, 21, 192–200.

Lerner, J. and Tirole, J. (2002): Some Simple Economics of Open Source, *Journal of Industrial Economics*, 50, 197–234.

Mustonen, M. (2003): Copyleft – The Economics of Linux and Other Open Source Software, *Information Economics and Policy*, 15, 99–121.

Page, S. E. (2006): Path Dependence, *Quarterly Journal of Political Science*, 1, 87–115.

Peitz, M. (2004): A Strategic Approach to Software Protection: Comment, *Journal of Economics & Management Strategy*, 13, 371–374.

Schiff, A. (2002): The Economics of Open Source Software: A Survey of the Early Literature, *Review of Network Economics*, 1, 66–74.

Schmidt, K. M. and Schnitzer, M. (2003): Public Subsidies for Open Source? Some Economic Policy Issues of the Software Market, *Harvard Journal of Law and Technology*, 16, 473–505.

Shy, O. (2001): *The Economics of Network Industries*, Cambridge: Cambridge University Press.

Shy, O. and Thisse, J.-F. (1999): A Strategic Approach to Software Protection, *Journal of Economics & Management Strategy*, 8, 163–190.

Varian, H. R. and Shapiro, C. (2007): Einführung von Linux im öffentlichen Sektor, in: Gehring and Lutterbeck (2007), 209–222.